Package 'SpatialExtremes'

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Title Modelling Spatial Extremes

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Depends R (>= 1.8.0)

Description This package proposes several approaches for spatial extremes modelling.

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URL http://spatialextremes.r-forge.r-project.org/

Archs i386, x86_64

R topics documented:

`anova` *Anova Tables*

Description

Computes analysis of deviance for objects of class ”maxstab”

Usage

```
## S3 method for class 'maxstab'
anova(object, object2, method = "RJ", square = "chol",
...)
## S3 method for class 'spatgev'
anova(object, object2, method = "RJ", square = "chol",
...)
```

Arguments

`object, object2`	Two objects of class 'maxstab' or 'spatgev'.
`method`	Character string. Must be one of "CB" or "RJ" for the Chandler and Bate or the Rotnitzky and Jewell approaches respectively. See function `profile`.
`square`	The choice for the matrix square root. This is only useful for the 'CB' method. Must be one of 'chol' (Cholesky) or 'svd' (Singular Value Decomposition).
`...`	Other options to be passed to the `anova` function.

Details

As "maxstab" objects are fitted using pairwise likelihood, the model is misspecified. As a consequence, the likelihood ratio statistic is no longer χ^2 distributed. To compute the anova table, we use the methodology proposed by Rotnitzky and Jewell to adjust the distribution of the likelihood ratio statistic.

Value

This function returns an object of class anova. These objects represent analysis-of-deviance tables.

Author(s)

Mathieu Ribatet

References

Chandler, R. E. and Bate, S. (2007) Inference for clustered data using the independence loglikelihood *Biometrika*, **94**, 167–183.

Rotnitzky, A. and Jewell, N. (1990) Hypothesis testing of regression parameters in semiparametric generalized linear models for cluster correlated data. *Biometrika* **77**, 485–497.

See Also

`fitmaxstab`, `fitspatgev`, `profile`, `TIC`

Examples

```
##Define the coordinates of each location
n.site <- 30
locations <- matrix(rnorm(2*n.site, sd = sqrt(.2)), ncol = 2)
colnames(locations) <- c("lon", "lat")

##Simulate a max-stable process - with unit Frechet margins
data <- rmaxstab(50, locations, cov.mod = "gauss", cov11 = 100, cov12 =
25, cov22 = 220)

##Now define the spatial model for the GEV parameters
param.loc <- -10 + 2 * locations[,2]
param.scale <- 5 + 2 * locations[,1] + locations[,2]^2
param.shape <- rep(0.2, n.site)

##Transform the unit Frechet margins to GEV
for (i in 1:n.site)
```

```
  data[,i] <- frech2gev(data[,i], param.loc[i], param.scale[i],
param.shape[i])

##Define three models for the GEV margins to be fitted
loc.form <- loc ~ lat
scale.form <- scale ~ lon + I(lat^2)
shape.form <- shape ~ lon

M0 <- fitspatgev(data, locations, loc.form, scale.form, shape.form)
M1 <- fitspatgev(data, locations, loc.form, scale.form, shape.form,
shapeCoeff2 = 0)

##Model selection
anova(M0, M1)
anova(M0, M1, method = "CB", square = "svd")
```

`condmap` *Produces a conditional 2D map from a fitted max-stable process*

Description

Produces a conditional 2D map from a fitted max-stable process.

Usage

```
condmap(fitted, fix.coord, x, y, covariates = NULL, ret.per1 = 100,
ret.per2 = ret.per1, col = terrain.colors(64), plot.contour = TRUE,
...)
```

Arguments

`fitted`	An object of class `maxstab`. Most often, it will be the output of the function `fitmaxstab`.
`fix.coord`	The spatial coordinates of the location from which the conditional quantile is computed.
`x,y`	Numeric vector defining the grid at which the levels are computed.
`covariates`	An array specifying the covariates at each grid point defined by `x` and `y`. If `NULL`, no covariate is needed. See `map` to see how to build it.
`ret.per1,ret.per2`	Numerics giving the return period for which the quantile map is plotted. See details.
`col`	A list of colors such as that generated by 'rainbow', 'heat.colors', 'topo.colors', 'terrain.colors' or similar functions.
`plot.contour`	Logical. If `TRUE` (default), contour lines are added to the plot.
`...`	Several arguments to be passed to the `image` function.

Details

The function solves the following equation:

$$\Pr\left[Z(x_2) > z_2 | Z(x_1) > z_1\right] = \frac{1}{T_2}$$

where $z_1 = -1/\log(1 - 1/T_1)$.

In other words, it computes, given that at location x_1 we exceed the level z_1, the levels which is expected to be exceeded in average every T_2 year.

Value

A plot. Additionally, a list with the details for plotting the map is returned invisibly.

Author(s)

Mathieu Ribatet

See Also

map, filled.contour, heatmap, heat.colors, topo.colors, terrain.colors, rainbow

Examples

```
##Define the coordinate of each location
n.site <- 30
locations <- matrix(runif(2*n.site, 0, 10), ncol = 2)
colnames(locations) <- c("lon", "lat")

##Simulate a max-stable process - with unit Frechet margins
data <- rmaxstab(50, locations, cov.mod = "whitmat", sill = 1, range =
2, smooth = 1)

##Now define the spatial model for the GEV parameters
param.loc <- -10 - 4 * locations[,1] + locations[,2]^2
param.scale <- 5 + locations[,2] + locations[,1]^2 / 10
param.shape <- rep(.2, n.site)

##Transform the unit Frechet margins to GEV
for (i in 1:n.site)
  data[,i] <- frech2gev(data[,i], param.loc[i], param.scale[i],
param.shape[i])

##Define a model for the GEV margins to be fitted
##shape ~ 1 stands for the GEV shape parameter is constant
##over the region
loc.form <- loc ~ lon + I(lat^2)
scale.form <- scale ~ lat + I(lon^2)
shape.form <- shape ~ 1

##  1- Fit a max-stable process
fitted <- fitmaxstab(data, locations, "whitmat", loc.form, scale.form,
                     shape.form, sill = 1, std.err.type = "none")
```

```
cond.coord <- c(5.1, 5.1)
condmap(fitted, cond.coord, seq(0, 10, length = 25), seq(0,10, length
 =25), ret.per1 = 100, ret.per2 = 1.5)
points(t(cond.coord), pch = "*", col = 2, cex = 2)
```

`condrgp` *Conditional simulation of Gaussian random fields*

Description

This function generates conditional simulation of Gaussian random fields from the simple kriging predictor.

Usage

```
condrgp(n, coord, data.coord, data, cov.mod = "powexp", mean = 0, nugget
= 0, sill = 1, range = 1, smooth = 1, grid = FALSE, control = list())
```

Arguments

`n`	Integer. The number of conditional simulations.
`coord`	A numeric vector or matrix specifying the coordinates where the process has to be generated. If `coord` is matrix, each row specifies one location.
`data.coord`	A numeric vector or matrix specifying the coordinates where the process is conditioned.
`data`	A numeric vector giving the conditioning observations.
`cov.mod`	A character string specifying the covariance function family. Must be one of "whitmat", "powexp", "cauchy" or "bessel" for the Whittle-Mater, the powered exponential, the Cauchy or Bessel covariance families.
`mean,nugget,sill,range,smooth`	The mean, nugget, sill, range and smooth of the Gaussian process.
`grid`	Logical. Does `coord` specifies a grid?
`control`	A named list passing options to the simulation method of Gaussian processes — see `rgp`.

Value

A list with components:

`coord`	The coordinates at which the process was simulated;
`cond.sim`	The simulated process;
`data.coord`	The coordinates of the conditioning locations;
`data`	The conditioning observations;
`cov.mod`	The covariance function family;
`grid`	Does `coord` specifies a grid?

Author(s)

Mathieu Ribatet

See Also

kriging, rgp.

Examples

```
## Several conditional simulations
n.site <- 50
n.sim <- 512

x.obs <- runif(n.site, -100, 100)
x.sim <- seq(-100, 100, length = n.sim)

data <- rgp(1, x.obs, "whitmat", sill = 1, range = 10, smooth = 0.75)

sim <- condrgp(5, x.sim, x.obs, data, "whitmat", sill = 1, range =
10, smooth = 0.75)

matplot(x.sim, t(sim$cond.sim),  type = "l", lty = 1, xlab = "x", ylab =
expression(Y[cond](x)))
points(x.obs, data, pch = 21, bg = 1)
title("Five conditional simulations")

## Comparison between one conditional simulations and the kriging
## predictor on a grid
x.obs <- matrix(runif(2 * n.site, -100, 100), ncol = 2)
x <- y <- seq(-100, 100, length = 100)
x.sim <- cbind(x, y)

data <- rgp(1, x.obs, "whitmat", sill = 1, range = 50, smooth = 0.75)

krig <- kriging(data, x.obs, x.sim, "whitmat", sill = 1, range = 50,
smooth = 0.75, grid = TRUE)
sim <- condrgp(1, x.sim, x.obs, data, "whitmat", sill = 1, range = 50,
smooth = 0.75, grid = TRUE)

z.lim <- range(c(sim$cond.sim, data, krig$krig.est))
breaks <- seq(z.lim[1], z.lim[2], length = 65)
col <- heat.colors(64)
idx <- as.numeric(cut(data, breaks))

op <- par(mfrow = c(1,2))
image(x, y, krig$krig.est, col = col, breaks = breaks)
points(x.obs, bg = col[idx], pch = 21)
title("Kriging predictor")
image(x, y, sim$cond.sim, col = col, breaks = breaks)
points(x.obs, bg = col[idx], pch = 21)
title("Conditional simulation")
## Note how the background colors of the above points matches the ones
## returned by the image function
par(op)
```

`covariance` *Defines and computes covariance functions*

Description

This function defines and computes several covariance function either from a fitted "max-stable" model; either by specifying directly the covariance parameters.

Usage

```
covariance(fitted, sill, range, smooth, smooth2 = NULL, cov.mod =
"whitmat", plot = TRUE, dist, xlab, ylab, ...)
```

Arguments

`fitted`	An object of class "maxstab". Most often this will be the output of the `fitmaxstab` function. May be missing if `sill`, `range`, `smooth` and `cov.mod` are given.
`sill,range,smooth,smooth2`	The sill, scale and smooth parameters for the covariance function. May be missing if `fitted` is given.
`cov.mod`	Character string. The name of the covariance model. Must be one of "whitmat", "cauchy", "powexp", "bessel" or "caugen" for the Whittle-Matern, Cauchy, Powered Exponential, Bessel and Generalized Cauchy models. May be missing if `fitted` is given.
`plot`	Logical. If `TRUE` (default) the covariance function is plotted.
`dist`	A numeric vector corresponding to the distance at which the covariance function should be evaluated. May be missing.
`xlab,ylab`	The x-axis and y-axis labels. May be missing.
`...`	Several option to be passed to the `plot` function.

Details

Currently, four covariance functions are defined: the Whittle-Matern, powered exponential (also known as "stable"), Cauchy and Bessel models. These covariance functions are defined as follows:

Whittle-Matern $\rho(h) = sill \frac{2^{1-smooth}}{\Gamma(smooth)} \left(\frac{h}{range}\right)^{smooth} K_{smooth}\left(\frac{h}{range}\right)$

Powered Exponential $\rho(h) = sill \exp\left[-\left(\frac{h}{range}\right)^{smooth}\right]$

Cauchy $\rho(h) = sill \left[1 + \left(\frac{h}{range}\right)^2\right]^{-smooth}$

Bessel $\rho(h) = sill \left(\frac{2range}{h}\right)^{smooth} \Gamma(smooth+1) J_{smooth}\left(\frac{h}{range}\right)$

Generalized Cauchy $\rho(h) = sill \left\{1 + \left(\frac{h}{range}\right)^{smooth2}\right\}^{-smooth/smooth2}$

where Γ is the gamma function, K_{smooth} and J_{smooth} are both modified Bessel functions of order $smooth$.

Value

This function returns the covariance function. Eventually, if `dist` is given, the covariance function is computed for each distance given by `dist`. If `plot = TRUE`, the covariance function is plotted.

Author(s)

Mathieu Ribatet

Examples

```
## 1- Calling covariance using fixed covariance parameters
covariance(sill = 1, range = 1, smooth = 0.5, cov.mod = "whitmat")
covariance(sill = 0.5, range = 1, smooth = 0.5, cov.mod = "whitmat",
  dist = seq(0,5, 0.2), plot = FALSE)

## 2- Calling covariance from a fitted model
##Define the coordinate of each location
n.site <- 30
locations <- matrix(runif(2*n.site, 0, 10), ncol = 2)
colnames(locations) <- c("lon", "lat")

##Simulate a max-stable process - with unit Frechet margins
data <- rmaxstab(30, locations, cov.mod = "whitmat", sill = 1, range =
3, smooth = 1)

##Fit a max-stable model
fitted <- fitmaxstab(data, locations, "whitmat", std.err.type = "none",
sill = 1)
covariance(fitted, ylim = c(0, 1))
covariance(sill = 1, range = 3, smooth = 1, cov.mod = "whitmat", add =
TRUE, col = 3)
title("Whittle-Matern covariance function")
legend("topright", c("Theo.", "Fitted"), lty = 1, col = c(3,1), inset =
.05)
```

---

cv *Estimates the penalty coefficient from the cross-validation criterion*

Description

Estimates the penalty coefficient from the cross-validation criterion.

Usage

```
cv(y, x, knots, degree, plot = TRUE, n.points = 150, ...)
```

Arguments

y	The response vector.
x	A vector/matrix giving the values of the predictor variable(s). If x is a matrix, each row corresponds to one observation.
knots	A vector givint the coordinates of the knots.
degree	The degree of the penalized smoothing spline.
plot	Logical. If TRUE (default), the evolution of the CV score as the penalty increases is plotted.
n.points	A numeric giving the number of CV computations needed to produce the plot.
...	Options to be passed to the nlm function.

Details

For every linear smoother e.g. $\hat{y} = S_\lambda y$, the cross-validation criterion consists in minimizing the following quantity:

$$CV(\lambda) = \sum_{i=1}^{n} \left(\frac{y_i - \hat{y}_i}{1 - S_{\lambda,ii}} \right)^2$$

where λ is the penalty coefficient, n the number of observations and $S_{\lambda,ii}$ the i-th diagonal element of the matrix S_λ.

Value

A list with components 'penalty', 'cv' and 'nlm.code' which give the location of the minimum, the value of the cross-validation criterion at that point and the code returned by the `link{nlm}` function - useful to assess for convergence issues.

Author(s)

Mathieu Ribatet

References

Ruppert, D. Wand, M.P. and Carrol, R.J. (2003) *Semiparametric Regression* Cambridge Series in Statistical and Probabilistic Mathematics.

See Also

`cv`

Examples

```
n <- 200
x <- runif(n)
fun <- function(x) sin(3 * pi * x)
y <- fun(x) + rnorm(n, 0, sqrt(0.4))
knots <- quantile(x, prob = 1:(n/4) / (n/4 + 1))
cv(y, x, knots = knots, degree = 3)
```

DIC *Deviance Information Criterion*

Description

This function computes the Deviance Information Criterion (DIC), and related quantities, which is a hierarchical modeling generalization of the Akaike Information Criterion. It is useful for Bayesian model selection.

Usage

```
DIC(object, ..., fun = "mean")
```

Arguments

`object`	An object of class `latent` — typically this will be the output of `latent`.
`...`	Optional arguments. Not implemented.
`fun`	A chararcter string giving the name of the function to be used to summarize the Markov chain. The default is to consider the posterior mean.

Details

The deviance is

$$D(\theta) = -2 \log \pi(y \mid \theta),$$

where y are the data, θ are the unknown parameters of the models and $\pi(y \mid \theta)$ is the likelihood function. Thus the expected deviance, a measure of how well the model fits the data, is given by

$$\overline{D} = \mathrm{E}_\theta[D(\theta)],$$

while the effective number of parameters is

$$p_D = \overline{D} - D(\theta^*),$$

where θ^* is point estimate of the posterior distribution, e.g., the posterior mean. Finally the DIC is given by

$$\mathrm{DIC} = p_D + \overline{D}.$$

In accordance with the AIC, models with smaller DIC should be preferred to models with larger DIC. Roughly speaking, differences of more than 10 might rule out the model with the higher DIC, differences between 5 and 10 are substantial.

Value

A vector of length 3 that returns the DIC, effective number of parameters `eNoP` and an estimate of the expected deviance `Dbar`.

Author(s)

Mathieu Ribatet

References

Spiegelhalter, D. J., Best, N. G., Carlin, B. P. and Van Der Linde, A. (2002) Bayesian measures of model complexity and fit. *Journal of the Royal Statistical Society: Series B* **64**, 583–639.

See Also

`AIC`

Examples

```
## Generate realizations from the model
n.site <- 15
n.obs <- 35
coord <- cbind(lon = runif(n.site, -10, 10), lat = runif(n.site, -10 , 10))

gp.loc <- rgp(1, coord, "powexp", sill = 4, range = 20, smooth = 1)
gp.scale <- rgp(1, coord, "powexp", sill = 0.4, range = 5, smooth = 1)
```

```
gp.shape <- rgp(1, coord, "powexp", sill = 0.01, range = 10, smooth = 1)

locs <- 26 + 0.5 * coord[,"lon"] + gp.loc
scales <- 10 + 0.2 * coord[,"lat"] + gp.scale
shapes <- 0.15 + gp.shape

data <- matrix(NA, n.obs, n.site)
for (i in 1:n.site)
  data[,i] <- rgev(n.obs, locs[i], scales[i], shapes[i])

loc.form <- y ~ lon
scale.form <- y ~ lat
shape.form <- y ~ 1

hyper <- list()
hyper$sills <- list(loc = c(1,8), scale = c(1,1), shape = c(1,0.02))
hyper$ranges <- list(loc = c(2,20), scale = c(1,5), shape = c(1, 10))
hyper$smooths <- list(loc = c(1,1/3), scale = c(1,1/3), shape = c(1, 1/3))
hyper$betaMeans <- list(loc = rep(0, 2), scale = c(9, 0), shape = 0)
hyper$betaIcov <- list(loc = solve(diag(c(400, 100))),
                       scale = solve(diag(c(400, 100))),
                       shape = solve(diag(c(10), 1, 1)))

## We will use an exponential covariance function so the jump sizes for
## the shape parameter of the covariance function are null.
prop <- list(gev = c(2.5, 1.5, 0.3), ranges = c(40, 20, 20), smooths = c(0,0,0))
start <- list(sills = c(4, .36, 0.009), ranges = c(24, 17, 16), smooths
              = c(1, 1, 1),  beta = list(loc = c(26, 0), scale = c(10, 0),
                                shape = c(0.15)))

mc <- latent(data, coord, loc.form = loc.form, scale.form = scale.form,
             shape.form = shape.form, hyper = hyper, prop = prop, start = start,
             n = 500)
DIC(mc)
```

`distance` *Computes distance between pairs of locations*

Description

This function computes euclidean distance or distance vector for each pair of a set of spatial locations.

Usage

```
distance(coord, vec = FALSE)
```

Arguments

`coord`	A matrix representing the coordinates of each locations. Each row corresponds to one location.
`vec`	Logical. If `FALSE` (default), the euclidean distance is computed. Otherwise, "distance vectors" are returned.

Value

If `vec = FALSE`, this function returns a vector that gives the euclidean distance for each pair of locations. Otherwise, this is a matrix where each column correspond to one dimension - i.e. longitude, latitude, ...

Author(s)

Mathieu Ribatet

See Also

`dist`

Examples

```
coords <- cbind(seq(0, 10, by = 2), seq(10, 0, by = -2))
distance(coords)
distance(coords, vec = TRUE)
```

---

extcoeff *Plots the extremal coefficient*

Description

Plots the extremal coefficient evolution as the coordinates evolves.

Usage

```
extcoeff(fitted, cov.mod, param, n = 200, xlab, ylab, ...)
```

Arguments

`fitted`	A object of class "`maxstab`". Most often, it will be the output of the function `fitmaxstab`. If missing, then `cov.mod` and `param` should be supplied.
`cov.mod`	A character string corresponding the the covariance model in the max-stable representation. Must be one of "gauss" for the Smith's model; or "whitmat", "cauchy" or "powexp" for the Whittle-Matern, the Cauchy and the Powered Exponential covariance family with the Schlather's model. May be missing if `fitted` is given.
`param`	Numeric vector of length 3. The parameters for the Smith's or Schlather model - i.e. c(cov11, cov12, cov22) or c(sill, range, smooth). Please respect this order.
`n`	Numeric. `n^2` corresponds to the total number of estimated extremal coefficients for the contour plot.
`xlab,ylab`	The x-axis and y-axis labels. May be missing.
`...`	Several options to be passed to the `contour` function.

Value

A plot.

Author(s)

Mathieu Ribatet

See Also

fitmaxstab

Examples

```
## 1- Random field generation
n.site <- 30
locations <- matrix(runif(2*n.site, 0, 10), ncol = 2)
colnames(locations) <- c("lon", "lat")

data <- rmaxstab(60, locations, cov.mod = "whitmat", sill = 1, range =
3, smooth = 1)

## 2- Fit a max-stable processes
schlather <- fitmaxstab(data, locations, "whitmat", sill = 1)

## 3- Plot the extremal coefficient
extcoeff(schlather)
```

---

fitcovariance *Estimates the covariance function for the Schlather's model*

Description

Estimates the covariance function for the Schlather's model using non-parametric estimates of the pairwise extremal coefficients.

Usage

```
fitcovariance(data, coord, cov.mod, marge = "emp", control = list(),
..., start, weighted = TRUE)
```

Arguments

data	A matrix representing the data. Each column corresponds to one location.
coord	A matrix that gives the coordinates of each location. Each row corresponds to one location.
cov.mod	A character string corresponding the the covariance model in the Schlather's model. Must be one of "whitmat", "cauchy", "powexp", "bessel" or "caugen" for the Whittle-Matern, the Cauchy, the Powered Exponential, the Bessel and the Generalized Cauchy correlation families.
marge	Character string specifying how margins are transformed to unit Frechet. Must be one of "emp", "frech" or "mle" - see function fitextcoeff.
control	The control arguments to be passed to the optim function.
...	Optional arguments to be passed to the optim function.

`start`	A named list giving the initial values for the parameters over which the weighted sum of square is to be minimized. If `start` is omitted the routine attempts to find good starting values.
`weighted`	Logical. Should weighted least squares be used?

Details

The fitting procedure is based on weighted least squares. More precisely, the fitting criteria is to minimize:

$$\sum_{i,j} \left(\frac{\tilde{\theta}_{i,j} - \hat{\theta}_{i,j}}{s_{i,j}} \right)^2$$

where $\tilde{\theta}_{i,j}$ is a non parametric estimate of the extremal coefficient related to location `i` and `j`, $\hat{\theta}_{i,j}$ is the fitted extremal coefficient derived from the Schlather's model and $s_{i,j}$ are the standard errors related to the estimates $\tilde{\theta}_{i,j}$.

Value

An object of class maxstab.

Author(s)

Mathieu Ribatet

References

Smith, R. L. (1990) Max-stable processes and spatial extremes. Unpublished manuscript.

See Also

`fitcovmat`, `fitmaxstab`, `fitextcoeff`

Examples

```
n.site <- 50
locations <- matrix(runif(2*n.site, 0, 10), ncol = 2)
colnames(locations) <- c("lon", "lat")

##Simulating a max-stable process using RandomFields
##This is the Schlather's approach
data <- rmaxstab(50, locations, cov.mod = "whitmat", sill = 1, range =
30, smooth = 1)

fitcovariance(data, locations, "whitmat")
```

`fitcovmat` *Estimates the covariance matrix for the Smith's model*

Description

Estimates the covariance matrix for the Smith's model using non-parametric estimates of the pairwise extremal coefficients.

Usage

```
fitcovmat(data, coord, marge = "emp", iso = FALSE, control = list(),
..., start, weighted = TRUE)
```

Arguments

`data`	A matrix representing the data. Each column corresponds to one location.
`coord`	A matrix that gives the coordinates of each location. Each row corresponds to one location.
`marge`	Character string specifying how margins are transformed to unit Frechet. Must be one of "emp", "frech" or "mle" - see function `fitextcoeff`.
`iso`	Logical. If `TRUE`, isotropy is supposed. Otherwise (default), anisotropy is allowed.
`control`	The control arguments to be passed to the `optim` function.
`...`	Optional arguments to be passed to the `optim` function.
`start`	A named list giving the initial values for the parameters over which the weighted sum of square is to be minimized. If `start` is omitted the routine attempts to find good starting values.
`weighted`	Logical. Should weighted least squares be used?

Details

The fitting procedure is based on weighted least squares. More precisely, the fitting criteria is to minimize:

$$\sum_{i,j} \left(\frac{\tilde{\theta}_{i,j} - \hat{\theta}_{i,j}}{s_{i,j}} \right)^2$$

where $\tilde{\theta}_{i,j}$ is a non parametric estimate of the extremal coefficient related to location `i` and `j`, $\hat{\theta}_{i,j}$ is the fitted extremal coefficient derived from the Smith's model and $s_{i,j}$ are the standard errors related to the estimates $\tilde{\theta}_{i,j}$.

Value

An object of class maxstab.

Author(s)

Mathieu Ribatet

References

Smith, R. L. (1990) Max-stable processes and spatial extremes. Unpublished manuscript.

See Also

fitcovariance, fitmaxstab, fitextcoeff

Examples

```
n.site <- 50
n.obs <- 100
locations <- matrix(runif(2*n.site, 0, 40), ncol = 2)
colnames(locations) <- c("lon", "lat")

## Simulate a max-stable process - with unit Frechet margins
data <- rmaxstab(50, locations, cov.mod = "gauss", cov11 = 200, cov12 =
0, cov22 = 200)

fitcovmat(data, locations)

##Force an isotropic model
fitcovmat(data, locations, iso = TRUE)
```

---

fitextcoeff *Non parametric estimators of the extremal coefficient function*

Description

Estimates non parametrically the extremal coefficient function.

Usage

```
fitextcoeff(data, coord, ..., estim = "ST", marge = "emp", prob = 0,
plot = TRUE, loess = TRUE, method = "BFGS", std.err = TRUE, xlab,
ylab, angles = NULL, identify = FALSE)
```

Arguments

data	A matrix representing the data. Each column corresponds to one location.
coord	A matrix that gives the coordinates of each location. Each row corresponds to one location.
...	Additional options to be passed to the plot function.
estim	Character string specifying the estimator to be used. Must be one of "ST" (Schlather and Tawn) or "Smith".
marge	Character string specifying how margins are transformed to unit Frechet. Must be one of "emp", "mle" or "none" - see Details
prob	The probability related to the threshold. Only useful with the ST estimator.
plot	Logical. If TRUE (default), the extremal coefficient function is plotted
loess	If TRUE (default), a local polynomial curve is plotted - see function loess.

`method`	The optimizer used when fitting the GEV distribution to data. See function `gevmle`.
`std.err`	Logical. If `TRUE`, standard errors are computed. Note that standard errors are not available with the "ST" estimator.
`xlab,ylab`	The x-axis and y-axis labels. May be missing.
`angles`	A numeric vector. A partition of the interval $(-\pi, \pi)$ to help detecting anisotropy.
`identify`	Logical. If `TRUE`, users can use the `identify` function to identify pairs of stations on the plot.

Details

During the estimation procedure, data need to be transformed to unit Frechet margins firts. This can be done in two different ways ; by using the empirical CDF or the GEV ML estimates.

If `marge = "emp"`, then the data are transformed using the following relation:

$$z_i = -\frac{1}{\log(F(y_i))}$$

where y_i are the observations available at location i, F is the empirical CDF and z_i are the observations transformed to unit Frechet scale.

If `marge = "mle"`, then the data are transformed using the MLE of the GEV distribution - see function `gev2frech`.

Lastly, if data are already supposed to be unit Frechet, then no transformation is performed if one passed the option `marge = "frech"`.

If `data` are already componentwise maxima, `prob` should be zero. Otherwise, users have to define a threshold z (large enough) where univariate extreme value arguments are relevant. We define `prob` such that $\Pr[Z \leq z] = prob$.

Value

Plots the extremal coefficient function and returns the points used for the plot. If `loess = TRUE`, the output is a list with argument "ext.coeff" and "loess".

Author(s)

Mathieu Ribatet

References

Schlather, M. and Tawn, J. A. (2003) A dependence measure for multivariate and spatial extreme values: Properties and inference. Biometrika **90**(1):139–156.

Smith, R. L. (1990) Max-stable processes and spatial extremes. Unpublished manuscript.

See Also

`madogram`

Examples

```
n.site <- 30
locations <- matrix(runif(2*n.site, 0, 10), ncol = 2)
colnames(locations) <- c("lon", "lat")

##Simulate a max-stable process - with unit Frechet margins
data <- rmaxstab(50, locations, cov.mod = "gauss", cov11 = 10, cov12 =
40, cov22 = 220)

##Plot the extremal coefficient function
op <- par(mfrow=c(1,2))
fitextcoeff(data, locations, estim = "Smith")
fitextcoeff(data, locations, angles = seq(-pi, pi, length = 4), estim = "Smith")
par(op)
```

---

fitmaxstab *Fits a max-stable process to data*

Description

This function fits max-stable processes to data using pairwise likelihood. Two max-stable characterisations are available: the Smith and Schlather representations.

Usage

```
fitmaxstab(data,  coord, cov.mod, loc.form, scale.form,
shape.form, marg.cov = NULL, temp.cov = NULL, temp.form.loc = NULL,
temp.form.scale = NULL, temp.form.shape = NULL, iso = FALSE, ...,
fit.marge = FALSE, warn = TRUE, method = "Nelder", start, control =
list(), weights = NULL, std.err.type = "score", corr = FALSE)
```

Arguments

`data`	A matrix representing the data. Each column corresponds to one location.
`coord`	A matrix that gives the coordinates of each location. Each row corresponds to one location.
`cov.mod`	A character string corresponding to the covariance model in the max-stable representation. Must be one of "gauss" for the Smith's model; "whitmat", "cauchy", "powexp", "bessel" or "caugen" for the Whittle-Matern, the Cauchy, the Powered Exponential, the Bessel and the Generalized Cauchy correlation families with the Schlather's model; "brown" for Brown-Resnick processes. The geometric Gaussian and Extremal-t models with a Whittle-Matern correlation function can be fitted by passing respectively "gwhitmat" or "twhitmat". Other correlation function families are considered in a similar way.
`loc.form, scale.form, shape.form`	R formulas defining the spatial linear model for the GEV parameters. May be missing. See section Details.
`marg.cov`	Matrix with named columns giving additional covariates for the GEV parameters. If `NULL`, no extra covariates are used.

`temp.cov` Matrix with names columns giving additional *temporal* covariates for the GEV parameters. If `NULL`, no temporal trend are assume for the GEV parameters — see section Details.

`temp.form.loc, temp.form.scale, temp.form.shape` R formulas defining the temporal trends for the GEV parameters. May be missing. See section Details.

`iso` Logical. If `TRUE` an isotropic model is fitted to data. Otherwise (default), anisotropy is allowed. Currently, this is only implemented for the Smith's model.

`...` Several arguments to be passed to the `optim`, `nlm` or `nlminb` functions. See section details.

`fit.marge` Logical. If `TRUE`, the GEV parameters are estimated pointwise or using the formulas given by `loc.form`, `scale.form` and `shape.form`. If `FALSE`, observations are supposed to be unit Frechet distributed. Note that when formulas are given, `fit.marge` is automatically set to `TRUE`.

`warn` Logical. If `TRUE` (default), users are warned if the log-likelihood is infinite at starting values and/or problems arised while computing the standard errors.

`method` The method used for the numerical optimisation procedure. Must be one of `BFGS`, `Nelder-Mead`, `CG`, `L-BFGS-B`, `SANN`, `nlm` or `nlminb`. See `optim` for details. Please note that passing `nlm` or `nlminb` will use the `nlm` or `nlminb` functions instead of `optim`.

`start` A named list giving the initial values for the parameters over which the pairwise likelihood is to be minimized. If `start` is omitted the routine attempts to find good starting values - but might fail.

`control` A list giving the control parameters to be passed to the `optim` function.

`weights` A numeric vector specifying the weights in the pairwise likelihood - and so has length the number of pairs. If `NULL` (default), no weighting scheme is used.

`std.err.type` Character string. Must be one of "none", "score", "grad". If "none", standard errors are not computed. Otherwise, standard errors are estimated using the sandwich estimates - see section Details.

`corr` Logical. If `TRUE` (non default), the asymptotic correlation matrix is computed.

Details

As spatial data often deal with a large number of locations, it is impossible to write analytically the joint distribution. Consequently, the fitting procedure substitutes the "full likelihood" for the pairwise likelihood.

Let define $L_{i,j}(x_{i,j}, \theta)$ the likelihood for site i and j, where $i = 1, \ldots, N-1$, $j = i+1, \ldots, N$, N is the number of site within the region and $x_{i,j}$ are the joint observations for site i and j. Then the pairwise likelihood $PL(\theta)$ is defined by:

$$\ell_P = \log PL(\theta) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \log L_{i,j}(x_{i,j}, \theta)$$

As pairwise likelihood is an approximation of the "full likelihood", standard errors cannot be computed directly by the inverse of the Fisher information matrix. Instead, a sandwich estimate must be used to account for model mispecification e.g.

$$\hat{\theta} \sim N(\theta, H^{-1} J H^{-1})$$

where H is the Fisher information matrix (computed from the misspecified model) and J is the variance of the score function.

H is easily estimated using the observed Hessian matrix given by the `optim` function. Estimation of J is much more difficult. Currently, we propose two different strategies to estimate J.

`grad` J is estimated from the gradient e.g. $J = \sum_{i=1}^{n} \left\{ \nabla \ell_P(\hat{\theta}; y_i) \nabla \ell_P(\hat{\theta}, y_i)^T \right\}$.

`score` J is estimated directly from the variance of the observed score function.

There are two different kind of covariates : "spatial" and "temporal".

A "spatial" covariate may have different values accross station but does not depend on time. For example the coordinates of the stations are obviously "spatial". These "spatial" covariates should be used with the `marg.cov` and `loc.form, scale.form, shape.form`.

A "temporal" covariates may have different values accross time but does not depend on space. For example the years where the annual maxima were recorded is "temporal". These "temporal" covariates should be used with the `temp.cov` and `temp.form.loc, temp.form.scale, temp.form.shape`.

As a consequence note that `marg.cov` must have K rows (K being the number of sites) while `temp.cov` must have n rows (n being the number of observations).

Value

This function returns a object of class `maxstab`. Such objects are list with components:

`fitted.values`	A vector containing the estimated parameters.
`std.err`	A vector containing the standard errors.
`fixed`	A vector containing the parameters of the model that have been held fixed.
`param`	A vector containing all parameters (optimised and fixed).
`deviance`	The (pairwise) deviance at the maximum pairwise likelihood estimates.
`corr`	The correlation matrix.
`convergence, counts, message`	Components taken from the list returned by `optim` - for the `mle` method.
`data`	The data analysed.
`model`	The max-stable characterisation used.
`fit.marge`	A logical that specifies if the GEV margins were estimated.
`cov.fun`	The estimated covariance function - for the Schlather model only.
`extCoeff`	The estimated extremal coefficient function.
`cov.mod`	The covariance model for the spatial structure.

Warning

When using reponse surfaces to model spatially the GEV parameters, the likelihood is pretty rough so that the general purpose optimization routines may fail. It is your responsability to check if the numerical optimization succeeded or not. I tried, as best as I can, to provide warning messages if the optimizers failed but in some cases, no warning will appear!

Author(s)

Mathieu Ribatet

References

Cox, D. R. and Reid, N. (2004) A note on pseudo-likelihood constructed from marginal densities. *Biometrika* **91**, 729–737.

Demarta, S. and McNeil, A. (2005) The t copula and Related Copulas *International Statistical Review* **73**, 111-129.

Gholam–Rezaee, M. (2009) Spatial extreme value: A composite likelihood. PhD Thesis. Ecole Polytechnique Federale de Lausanne.

Kabluchko, Z., Schlather, M. and de Haan, L. (2009) Stationary max-stable fields associated to negative definite functions *Annals of Probability* **37**:5, 2042–2065.

Padoan, S. A. (2008) Computational Methods for Complex Problems in Extreme Value Theory. PhD Thesis. University of Padova.

Padoan, S. A., Ribatet, M. and Sisson, S. A. (2010) Likelihood-based inference for max-stable processes. *Journal of the American Statistical Association (Theory and Methods)* **105**:489, 263-277.

Schlather, M. (2002) Models for Stationary Max-Stable Random Fields. *Extremes* **5**:1, 33–44.

Smith, R. L. (1990) Max-stable processes and spatial extremes. Unpublished.

Examples

```
## Not run:
##Define the coordinate of each location
n.site <- 30
locations <- matrix(runif(2*n.site, 0, 10), ncol = 2)
colnames(locations) <- c("lon", "lat")

##Simulate a max-stable process - with unit Frechet margins
data <- rmaxstab(40, locations, cov.mod = "whitmat", sill = 1, range = 30,
smooth = 0.5)

##Now define the spatial model for the GEV parameters
param.loc <- -10 + 2 * locations[,2]
param.scale <- 5 + 2 * locations[,1] + locations[,2]^2
param.shape <- rep(0.2, n.site)

##Transform the unit Frechet margins to GEV
for (i in 1:n.site)
  data[,i] <- frech2gev(data[,i], param.loc[i], param.scale[i],
param.shape[i])

##Define a model for the GEV margins to be fitted
##shape ~ 1 stands for the GEV shape parameter is constant
##over the region
loc.form <- loc ~ lat
scale.form <- scale ~ lon + I(lat^2)
shape.form <- shape ~ 1

##Fit a max-stable process using the Schlather's model
fitmaxstab(data, locations, "whitmat", loc.form, scale.form,
           shape.form)

## Model without any spatial structure for the GEV parameters
## Be careful this could be *REALLY* time consuming
fitmaxstab(data, locations, "whitmat")
```

```
##  Fixing the smooth parameter of the Whittle-Matern family
##  to 0.5 - e.g. considering exponential family. We suppose the data
##  are unit Frechet here.
fitmaxstab(data, locations, "whitmat", smooth = 0.5, fit.marge = FALSE)

##  Fitting a penalized smoothing splines for the margins with the
##     Smith's model
data <- rmaxstab(40, locations, cov.mod = "gauss", cov11 = 100, cov12 =
                 25, cov22 = 220)

##     And transform it to ordinary GEV margins with a non-linear
##     function
fun <- function(x)
  2 * sin(pi * x / 4) + 10
fun2 <- function(x)
  (fun(x) - 7 ) / 15

param.loc <- fun(locations[,2])
param.scale <- fun(locations[,2])
param.shape <- fun2(locations[,1])

##Transformation from unit Frechet to common GEV margins
for (i in 1:n.site)
  data[,i] <- frech2gev(data[,i], param.loc[i], param.scale[i],
param.shape[i])

##Defining the knots, penalty, degree for the splines
n.knots <- 5
knots <- quantile(locations[,2], prob = 1:n.knots/(n.knots+1))
knots2 <- quantile(locations[,1], prob = 1:n.knots/(n.knots+1))

##Be careful the choice of the penalty (i.e. the smoothing parameter)
##may strongly affect the result Here we use p-splines for each GEV
##parameter - so it's really CPU demanding but one can use 1 p-spline
##and 2 linear models.
##A simple linear model will be clearly faster...
loc.form <- y ~ rb(lat, knots = knots, degree = 3, penalty = .5)
scale.form <- y ~ rb(lat, knots = knots, degree = 3, penalty = .5)
shape.form <- y ~ rb(lon, knots = knots2, degree = 3, penalty = .5)

fitted <- fitmaxstab(data, locations, "gauss", loc.form, scale.form, shape.form,
                     control = list(ndeps = rep(1e-6, 24), trace = 10),
                     std.err.type = "none", method = "BFGS")
fitted
op <- par(mfrow=c(1,3))
plot(locations[,2], param.loc, col = 2, ylim = c(7, 14),
     ylab = "location parameter", xlab = "latitude")
plot(fun, from = 0, to = 10, add = TRUE, col = 2)
points(locations[,2], predict(fitted)[,"loc"], col = "blue", pch = 5)
new.data <- cbind(lon = seq(0, 10, length = 100), lat = seq(0, 10, length = 100))
lines(new.data[,1], predict(fitted, new.data)[,"loc"], col = "blue")
legend("topleft", c("true values", "predict. values", "true curve", "predict. curve"),
       col = c("red", "blue", "red", "blue"), pch = c(1, 5, NA, NA), inset = 0.05,
       lty = c(0, 0, 1, 1), ncol = 2)

plot(locations[,2], param.scale, col = 2, ylim = c(7, 14),
```

```
     ylab = "scale parameter", xlab = "latitude")
plot(fun, from = 0, to = 10, add = TRUE, col = 2)
points(locations[,2], predict(fitted)[,"scale"], col = "blue", pch = 5)
lines(new.data[,1], predict(fitted, new.data)[,"scale"], col = "blue")
legend("topleft", c("true values", "predict. values", "true curve", "predict. curve"),
       col = c("red", "blue", "red", "blue"), pch = c(1, 5, NA, NA), inset = 0.05,
       lty = c(0, 0, 1, 1), ncol = 2)

plot(locations[,1], param.shape, col = 2,
     ylab = "shape parameter", xlab = "longitude")
plot(fun2, from = 0, to = 10, add = TRUE, col = 2)
points(locations[,1], predict(fitted)[,"shape"], col = "blue", pch = 5)
lines(new.data[,1], predict(fitted, new.data)[,"shape"], col = "blue")
legend("topleft", c("true values", "predict. values", "true curve", "predict. curve"),
       col = c("red", "blue", "red", "blue"), pch = c(1, 5, NA, NA), inset = 0.05,
       lty = c(0, 0, 1, 1), ncol = 2)
par(op)

## End(Not run)
```

`fitspatgev` *MLE for a spatial GEV model*

Description

This function derives the MLE of a spatial GEV model.

Usage

```
fitspatgev(data, covariables, loc.form, scale.form, shape.form,
temp.cov = NULL, temp.form.loc = NULL, temp.form.scale = NULL,
temp.form.shape = NULL, ..., start, control = list(maxit = 10000),
method = "Nelder", std.err.type = "score", warn = TRUE, corr = FALSE)
```

Arguments

`data`	A matrix representing the data. Each column corresponds to one location.
`covariables`	Matrix with named columns giving required covariates for the GEV parameter models.
`loc.form, scale.form, shape.form`	R formulas defining the spatial models for the GEV parameters. See section Details.
`temp.cov`	Matrix with names columns giving additional *temporal* covariates for the GEV parameters. If `NULL`, no temporal trend are assume for the GEV parameters — see section Details.
`temp.form.loc, temp.form.scale, temp.form.shape`	R formulas defining the temporal trends for the GEV parameters. May be missing. See section Details.
`start`	A named list giving the initial values for the parameters over which the pairwise likelihood is to be minimized. If `start` is omitted the routine attempts to find good starting values - but might fail.

...	Several arguments to be passed to the optim functions. See section details.
control	The control argument to be passed to the optim function.
method	The method used for the numerical optimisation procedure. Must be one of BFGS, Nelder-Mead, CG, L-BFGS-B or SANN. See optim for details.
std.err.type	Character string. Must be one of "score", "grad" or "none". If none, no standard errors are computed.
warn	Logical. If TRUE (default), users will be warned if the starting values lie in a zero density region.
corr	Logical. If TRUE, the asymptotic correlation matrix is computed.

Details

A kind of "spatial" GEV model can be defined by using response surfaces for the GEV parameters. For instance, the GEV location parameters are defined through the following equation:

$$\mu = X_\mu \beta_\mu$$

where X_μ is the design matrix and β_μ is the vector parameter to be estimated. The GEV scale and shape parameters are defined accordingly to the above equation.

The log-likelihood for the GEV spatial model is consequently defined as follows:

$$\ell(\beta) = \sum_{i=1}^{n.site} \sum_{j=1}^{n.obs} \log f(y_{i,j}; \theta_i)$$

where θ_i is the vector of the GEV parameters for the i-th site.

Most often, there will be some dependence between stations. However, it can be seen from the log-likelihood definition that we supposed that the stations are mutually independent. Consequently, to get reliable standard error estimates, these standard errors are estimated with their sandwich estimates.

There are two different kind of covariates : "spatial" and "temporal".

A "spatial" covariate may have different values accross station but does not depend on time. For example the coordinates of the stations are obviously "spatial". These "spatial" covariates should be used with the marg.cov and loc.form, scale.form, shape.form.

A "temporal" covariates may have different values accross time but does not depend on space. For example the years where the annual maxima were recorded is "temporal". These "temporal" covariates should be used with the temp.cov and temp.form.loc, temp.form.scale, temp.form.shape.

As a consequence note that marg.cov must have K rows (K being the number of sites) while temp.cov must have n rows (n being the number of observations).

Value

An object of class spatgev. Namely, this is a list with the following arguments:

fitted.values	The parameter estimates.
param	All the parameters e.g. parameter estimates and fixed parameters.
std.err	The standard errors.

`var.cov` The asymptotic MLE variance covariance matrix.

`counts,message,convergence`
Some information about the optimization procedure.

`logLik,deviance`
The log-likelihood and deviance values.

`loc.form, scale.form, shape.form`
The formulas defining the spatial models for the GEV parameters.

`covariables` The covariables used for the spatial models.

`ihessian` The inverse of the Hessian matrix of the negative log-likelihood.

`jacobian` The variance covariance matrix of the score.

Author(s)

Mathieu Ribatet

Examples

```
## 1- Simulate a max-stable random field
n.site <- 35
locations <- matrix(runif(2*n.site, 0, 10), ncol = 2)
colnames(locations) <- c("lon", "lat")
data <- rmaxstab(50, locations, cov.mod = "whitmat", sill = 1, range =
  3, smooth = 0.5)

## 2- Transformation to non unit Frechet margins
param.loc <- -10 + 2 * locations[,2]
param.scale <- 5 + 2 * locations[,1]
param.shape <- rep(0.2, n.site)
for (i in 1:n.site)
  data[,i] <- frech2gev(data[,i], param.loc[i], param.scale[i],
param.shape[i])

## 3- Fit a ''spatial GEV'' mdoel to data with the following models for
##    the GEV parameters
form.loc <- loc ~ lat
form.scale <- scale ~ lon
form.shape <- shape ~ 1

fitspatgev(data, locations, form.loc, form.scale, form.shape)
```

---

fmadogram *Computes the F-madogram*

Description

Computes the F-madogram for max-stable processes.

Usage

```
fmadogram(data, coord, fitted, n.bins, which = c("mado", "ext"), xlab,
ylab, col = c(1, 2), angles = NULL, marge = "emp", add = FALSE, xlim =
c(0, max(dist)), ...)
```

Arguments

data	A matrix representing the data. Each column corresponds to one location.
coord	A matrix that gives the coordinates of each location. Each row corresponds to one location.
fitted	An object of class maxstab - usually the output of the fitmaxstab function. May be missing.
n.bins	The number of bins to be used. If missing, pairwise F-madogram estimates will be computed.
which	A character vector of maximum size 2. It specifies if the madogram and/or the extremal coefficient functions have to be plotted.
xlab,ylab	The x-axis and y-axis labels. May be missing. Note that ylab must have the same length has which.
col	The colors used for the points and optionnaly the fitted curve.
angles	A numeric vector. A partition of the interval $(-\pi, \pi)$ to help detecting anisotropy.
marge	Character string. If 'emp', the probabilities of non exceedances are estimated using the empirical CDF. If 'mle' (default), maximum likelihood estimates are used.
add	Logical. If TRUE, the plot is added to the current figure; otherwhise (default) a new plot is computed.
xlim	A numeric vector of length 2 specifying the x coordinate range.
...	Additional options to be passed to the plot function.

Details

Let $Z(x)$ be a stationary process. The F-madogram is defined as follows:

$$\nu(h) = \frac{1}{2}\mathrm{E}\left[|F(Z(x+h)) - F(Z(x))|\right]$$

The extremal coefficient $\theta(h)$ satisfies:

$$\theta(h) = \frac{1 + 2\nu(h)}{1 - 2\nu(h)}$$

Value

A graphic and (invisibly) a matrix with the lag distances, the F-madogram and extremal coefficient estimates.

Author(s)

Mathieu Ribatet

References

Cooley, D., Naveau, P. and Poncet, P. (2006) Variograms for spatial max-stable random fields. *Dependence in Probability and Statistics*, 373–390.

See Also

madogram, lmadogram

Examples

```
n.site <- 15
locations <- matrix(runif(2*n.site, 0, 10), ncol = 2)
colnames(locations) <- c("lon", "lat")

##Simulate a max-stable process - with unit Frechet margins
data <- rmaxstab(40, locations, cov.mod = "whitmat", sill = 1, range = 1,
smooth = 2)

##Compute the F-madogram
fmadogram(data, locations)

##Compare the F-madogram with a fitted max-stable process
fitted <- fitmaxstab(data, locations, "whitmat", sill = 1)
fmadogram(fitted = fitted, which = "ext")
```

---

gcv *Estimates the penalty coefficient from the generalized cross-validation*

Description

Estimates the penalty coefficient from the generalized cross-validation criterion.

Usage

```
gcv(y, x, knots, degree, plot = TRUE, n.points = 150, ...)
```

Arguments

y	The response vector.
x	A vector/matrix giving the values of the predictor variable(s). If x is a matrix, each row corresponds to one observation.
knots	A vector givint the coordinates of the knots.
degree	The degree of the penalized smoothing spline.
plot	Logical. If TRUE (default), the evolution of the CV score as the penalty increases is plotted.
n.points	A numeric giving the number of CV computations needed to produce the plot.
...	Options to be passed to the nlm function.

Details

For every linear smoother e.g. $\hat{y} = S_\lambda y$, the cross-validation criterion consists in minimizing the following quantity:

$$GCV(\lambda) = \frac{n||y - \hat{y}||^2}{(n - tr(S_\lambda))^2}$$

where λ is the penalty coefficient, n the number of observations and $tr(S_\lambda)$ is the trace of the matrix S_λ.

Value

A list with components 'penalty', 'gcv' and 'nlm.code' which give the location of the minimum, the value of the cross-validation criterion at that point and the code returned by the `link{nlm}` function - useful to assess for convergence issues.

Author(s)

Mathieu Ribatet

References

Ruppert, D. Wand, M.P. and Carrol, R.J. (2003) *Semiparametric Regression* Cambridge Series in Statistical and Probabilistic Mathematics.

See Also

`cv`

Examples

```
n <- 200
x <- runif(n)
fun <- function(x) sin(3 * pi * x)
y <- fun(x) + rnorm(n, 0, sqrt(0.4))
knots <- quantile(x, prob = 1:(n/4) / (n/4 + 1))
gcv(y, x, knots = knots, degree = 3)
```

Generalized Extreme Value Distribution

The Generalized Extreme Value Distribution

Description

Density, distribution function, quantile function and random generation for the GP distribution with location equal to 'loc', scale equal to 'scale' and shape equal to 'shape'.

Usage

```
rgev(n, loc = 0, scale = 1, shape = 0)
pgev(q, loc = 0, scale = 1, shape = 0, lower.tail = TRUE)
qgev(p, loc = 0, scale = 1, shape = 0, lower.tail = TRUE)
dgev(x, loc = 0, scale = 1, shape = 0, log = FALSE)
```

Arguments

`x, q`	vector of quantiles.
`p`	vector of probabilities.
`n`	number of observations.
`loc`	vector of the location parameters.
`scale`	vector of the scale parameters.

`shape`	a numeric of the shape parameter.
`lower.tail`	logical; if TRUE (default), probabilities are $\Pr[X \leq x]$, otherwise, $\Pr[X > x]$.
`log`	logical; if TRUE, probabilities p are given as log(p).

Value

If 'loc', 'scale' and 'shape' are not specified they assume the default values of '0', '1' and '0', respectively.

The GEV distribution function for loc = u, scale = σ and shape = ξ is

$$G(x) = \exp\left[-\left\{1 + \xi\frac{x-u}{\sigma}\right\}^{-1/\xi}\right]$$

for $1 + \xi(x-u)/\sigma > 0$ and $x > u$, where $\sigma > 0$. If $\xi = 0$, the distribution is defined by continuity corresponding to the Gumbel distribution.

Examples

```
dgev(0.1)
rgev(100, 1, 2, 0.2)
qgev(seq(0.1, 0.9, 0.1), 1, 0.5, -0.2)
pgev(12.6, 2, 0.5, 0.1)
```

`Generalized Pareto Distribution`

The Generalized Pareto Distribution

Description

Density, distribution function, quantile function and random generation for the GP distribution with location equal to 'loc', scale equal to 'scale' and shape equal to 'shape'.

Usage

```
rgpd(n, loc = 0, scale = 1, shape = 0)
pgpd(q, loc = 0, scale = 1, shape = 0, lower.tail = TRUE, lambda = 0)
qgpd(p, loc = 0, scale = 1, shape = 0, lower.tail = TRUE, lambda = 0)
dgpd(x, loc = 0, scale = 1, shape = 0, log = FALSE)
```

Arguments

`x, q`	vector of quantiles.
`p`	vector of probabilities.
`n`	number of observations.
`loc`	vector of the location parameters.
`scale`	vector of the scale parameters.
`shape`	a numeric of the shape parameter.
`lower.tail`	logical; if TRUE (default), probabilities are $\Pr[X \leq x]$, otherwise, $\Pr[X > x]$.
`log`	logical; if TRUE, probabilities p are given as log(p).
`lambda`	a single probability - see the "value" section.

Value

If 'loc', 'scale' and 'shape' are not specified they assume the default values of '0', '1' and '0', respectively.

The GP distribution function for loc = u, scale = σ and shape = ξ is

$$G(x) = 1 - \left[1 + \frac{\xi(x-u)}{\sigma}\right]^{-1/\xi}$$

for $1 + \xi(x-u)/\sigma > 0$ and $x > u$, where $\sigma > 0$. If $\xi = 0$, the distribution is defined by continuity corresponding to the exponential distribution.

By definition, the GP distribution models exceedances above a threshold. In particular, the G function is a suited candidate to model

$$\Pr[X \geq x | X > u] = 1 - G(x)$$

for u large enough.

However, it may be usefull to model the "non conditional" quantiles, that is the ones related to $\Pr[X \leq x]$. Using the conditional probability definition, one have :

$$\Pr[X \geq x] = (1 - \lambda)\left(1 + \xi\frac{x-u}{\sigma}\right)^{-1/\xi}$$

where $\lambda = \Pr[X \leq u]$.

When $\lambda = 0$, the "conditional" distribution is equivalent to the "non conditional" distribution.

Examples

```
dgpd(0.1)
rgpd(100, 1, 2, 0.2)
qgpd(seq(0.1, 0.9, 0.1), 1, 0.5, -0.2)
pgpd(12.6, 2, 0.5, 0.1)
##for non conditional quantiles
qgpd(seq(0.9, 0.99, 0.01), 1, 0.5, -0.2, lambda = 0.9)
pgpd(2.6, 2, 2.5, 0.25, lambda = 0.5)
```

---

gev2frech *Transforms GEV data to unit Frechet ones and vice versa*

Description

Transforms GEV data to unit Frechet ones and vice versa

Usage

```
gev2frech(x, loc, scale, shape, emp = FALSE)
frech2gev(x, loc, scale, shape)
```

Arguments

x	The data to be transformed to unit Frechet or ordinary GEV margins
loc, scale, shape	The location, scale and shape parameters of the GEV.
emp	Logical. If TRUE, data are transformed to unit Frechet margins using the empirical CDF.

Details

If Y is a random variable with a GEV distribution with location μ, scale σ and shape ξ. Then,

$$Z = \left[1 + \xi \frac{Y - \mu}{\sigma}\right]_+^{1/\xi}$$

is unit Frechet distributed - where $x_+ = \max(0, x)$.

If Z is a unit Frechet random variable. Then,

$$Y = \mu + \sigma \frac{Z_+^{\xi} - 1}{\xi}$$

is unit GEV distributed with location, scale and shape parameters equal to μ, σ and ξ respectively.

Value

Returns a numeric vector with the same length of x

Author(s)

Mathieu Ribatet

Examples

```
x <- c(2.2975896, 1.6448808, 1.3323833, -0.4464904, 2.2737603,
    -0.2581876, 9.5184398, -0.5899699, 0.4974283, -0.8152157)
y <- gev2frech(x, 1, 2, .2)
y
frech2gev(y, 1, 2, .2)
```

---

kriging *Simple kriging interpolation*

Description

This function interpolates a zero mean Gaussian random field using the simple kriging predictor.

Usage

```
kriging(data, data.coord, krig.coord, cov.mod = "whitmat", sill, range,
smooth, smooth2 = NULL, grid = FALSE, only.weights = FALSE)
```

Arguments

data	A numeric vector or matrix. If data is a matrix then the simple kriging predictor is given for each realisation, i.e., each row of data.
data.coord	A numeric vector or matrix specifying the coordinates of the observed data. If data.coord is a matrix, each row must corresponds to one location.
krig.coord	A numeric vector or matrix specifying the coordinates where the kriging predictor has to be computed. If krig.coord is a matrix, each row must correspond to one location.
cov.mod	A character string specifying the covariance function family. Must be one of "whitmat", "powexp", "cauchy", "bessel" or "caugen" for the Whittle-Matern, the powered exponential, the Cauchy, the Bessel or the generalized Cauchy covariance families.
sill,range,smooth,smooth2	Numerics specifiying the sill, range, smooth and, if any, the second smooth parameters of the covariance function.
grid	Logical. Does krig.coord specifies a grid?
only.weights	Logical. Should only the kriging weights be computed? If FALSE, the kriging predictor isn't computed.

Value

A list with components

coord	The coordinates where the kriging predictor has been computed;
krig.est	The kriging predictor estimates;
grid	Does coord define a grid?;
weights	A matrix giving the kriging weights: each column corresponds to one prediction location.

Author(s)

Mathieu Ribatet

References

Chiles, J.-P. and Delfiner, P. (1999) *Geostatistics, Modeling Spatial Uncertainty* Wiley Series in Probability and Statistics.

See Also

condrgp, rgp, covariance.

Examples

```
## Kriging from a single realisation
n.site <- 50
n.pred <- 512

x.obs <- runif(n.site, -100, 100)
x.pred <- seq(-100, 100, length = n.pred)

data <- rgp(1, x.obs, "whitmat", sill = 1, range = 10, smooth = 0.75)
```

```
krig <- kriging(data, x.obs, x.pred, "whitmat", sill = 1, range = 10,
smooth = 0.75)

plot(krig$coord, krig$krig.est, type = "l", xlab = "x", ylab =
expression(hat(Y)(x)))
points(x.obs, data, col = 2, pch = 21, bg = 2)

## Kriging from several realisations
n.real <- 3
data <- rgp(n.real, x.obs, "whitmat", sill = 1, range = 10, smooth = 0.75)

krig <- kriging(data, x.obs, x.pred, "whitmat", sill = 1, range = 10,
smooth = 0.75)

matplot(krig$coord, t(krig$krig.est), type = "l", xlab = "x", ylab =
expression(hat(Y)(x)), lty = 1)
matpoints(x.obs, t(data), pch = 21, col = 1:n.real, bg = 1:n.real)
title("Three kriging predictors in one shot")

## Two dimensional kriging on a grid
x.obs <- matrix(runif(2 * n.site, -100, 100), ncol = 2)
x <- y <- seq(-100, 100, length = 100)
x.pred <- cbind(x, y)

data <- rgp(1, x.obs, "whitmat", sill = 1, range = 10, smooth = 0.75)

krig <- kriging(data, x.obs, x.pred, "whitmat", sill = 1, range = 10,
smooth = 0.75, grid = TRUE)

z.lim <- range(c(data, krig$krig.est))
breaks <- seq(z.lim[1], z.lim[2], length = 65)
col <- heat.colors(64)
idx <- as.numeric(cut(data, breaks))

image(x, y, krig$krig.est, col = col, breaks = breaks)
points(x.obs, bg = col[idx], pch = 21)
## Note how the background colors of the above points matches the ones
## returned by the image function
```

`latent` *Bayesian hierarchical models for spatial extremes*

Description

This function generates a Markov chain from a Bayesian hierarchical model for block maxima assuming conditional independence.

Usage

```
latent(data, coord, cov.mod = "powexp", loc.form, scale.form,
shape.form, marg.cov = NULL, hyper, prop, start, n = 5000, thin = 1,
burn.in = 0)
```

Arguments

`data`	A matrix representing the data. Each column corresponds to one location.
`coord`	A matrix that gives the coordinates of each location. Each row corresponds to one location.
`cov.mod`	A character string corresponding to the covariance model for the Gaussian latent processes. Must be one of "gauss" for the Smith's model; "whitmat", "cauchy", "powexp" or "bessel" or for the Whittle-Matern, the Cauchy, the Powered Exponential and the Bessel correlation families.
`loc.form, scale.form, shape.form`	R formulas defining the spatial linear model for the mean of the latent processes.
`marg.cov`	Matrix with named columns giving additional covariates for the latent processes means. If `NULL`, no extra covariates are used.
`hyper`	A named list specifying the hyper-parameters — see Details.
`prop`	A named list specifying the jump sizes when a Metropolis–Hastings move is needed — see Details.
`start`	A named list specifying the starting values — see Details.
`n`	The effective length of the simulated Markov chain i.e., once the burnin period has been discarded and after thinning.
`thin`	An integer specifying the thinning length. The default is 1, i.e., no thinning.
`burn.in`	An integer specifying the burnin period. The default is 0, i.e., no burnin.

Details

This function generates a Markov chain from the following model. For each $x \in R^d$, suppose that $Y(x)$ is GEV distributed whose parameters $\{\mu(x), \sigma(x), \xi(x)\}$ vary smoothly for $x \in R^d$ according to a stochastic process $S(x)$. We assume that the processes for each GEV parameters are mutually independent Gaussian processes. For instance, we take for the location parameter $\mu(x)$

$$\mu(x) = f_{\mu(x)}(x; \beta_\mu) + S_\mu(x; \alpha_\mu, \lambda_\mu, \kappa_\mu)$$

where f_μ is a deterministic function depending on regression parameters β_μ, and S_μ is a zero mean, stationary Gaussian process with a prescribed covariance function with sill α_μ, range λ_μ and shape parameters κ_μ. Similar formulations for the scale $\sigma(x)$ and the shape $\xi(x)$ parameters are used. Then conditional on the values of the three Gaussian processes at the sites $(x_1, \ldots, x_K)$, the maxima are assumed to follow GEV distributions

$$Y_i(x_j) \mid \{\mu(x_j), \sigma(x_j), \xi(x_j)\} \sim \text{GEV}\{\mu(x_j), \sigma(x_j), \xi(x_j)\},$$

independently for each location $(x_1, \ldots, x_K)$.

A joint prior density must be defined for the sills, ranges, shapes parameters of the covariance functions as well as for the regression parameters β_μ, β_σ and β_ξ. Conjugate priors are used whenever possible, taking independent inverse Gamma and multivariate normal distributions for the sills and the regression parameters. No conjugate prior exist for λ and κ, for wich a Gamma distribution is assumed.

Consequently `hyper` is a named list with named components

sills A list with three components named 'loc', 'scale' and 'shape' each of these is a 2-length vector specifying the shape and the scale of the inverse Gamma prior distribution for the sill parameter of the covariance functions;

ranges A list with three components named 'loc', 'scale' and 'shape' each of these is a 2-length vector specifying the shape and the scale of the Gamma prior distribution for the range parameter of the covariance functions.

smooths A list with three components named 'loc', 'scale' and 'shape' each of these is a 2-length vector specifying the shape and the scale of the Gamma prior distribution for the shape parameter of the covariance functions;

betaMean A list with three components named 'loc', 'scale' and 'shape' each of these is a vector specifying the mean vector of the multivariate normal prior distribution for the regression parameters;

betaIcov A list with three components named 'loc', 'scale' and 'shape' each of these is a matrix specifying the inverse of the covariance matrix of the multivariate normal prior distribution for the regression parameters.

As no conjugate prior exists for the GEV parameters and the range and shape parameters of the covariance functions, Metropolis–Hastings steps are needed. The proposals θ_{prop} are drawn from a proposal density $q(\cdot \mid \theta_{cur}, s)$ where θ_{cur} is the current state of the parameter and s is a parameter of the proposal density to be defined. These proposals are driven by `prop` which is a list with three named components

gev A vector of length 3 specifying the jump sizes for the location, scale and shape GEV parameters — $\theta_{prop} = \theta_{cur} + s(U - 0.5)$, $U \sim U(0, 1)$;

ranges A vector of length 3 specifying the jump sizes for the range parameters of the covariance functions — $q(\cdot|\theta_{cur}, s)$ is the log-normal density with mean θ_{cur} and standard deviation s both on the log-scale;

smooths A vector of length 3 specifying the jump sizes for the shape parameters of the covariance functions — $q(\cdot|\theta_{cur}, s)$ is the log-normal density with mean θ_{cur} and standard deviation s both on the log-scale.

If one want to held fixed a parameter this can be done by setting a null jump size then the parameter will be held fixed to its starting value.

Finally `start` must be a named list with 4 named components

sills A vector of length 3 specifying the starting values for the sill of the covariance functions;

ranges A vector of length 3 specifying the starting values for the range of the covariance functions;

smooths A vector of length 3 specifying the starting values for the shape of the covariance functions;

beta A named list with 3 components 'loc', 'scale' and 'shape' each of these is a numeric vector specifying the starting values for the regression coefficients.

Value

A list

Warning

This function can be time consuming and makes an intensive use of BLAS routines so it is (much!) faster if you have an optimized BLAS.

The starting values will never be stored in the generated Markov chain even when `burn.in=0`.

Note

If you want to analyze the convergence ans mixing properties of the Markov chain, it is recommended to use the library **coda**.

Author(s)

Mathieu Ribatet

References

Banerjee, S., Carlin, B. P., and Gelfand, A. E. (2004). Hierarchical Modeling and Analysis for Spatial Data. Chapman & Hall/CRC, New York.

Casson, E. and Coles, S. (1999) Spatial regression models for extremes. *Extremes* **1**,449–468.

Cooley, D., Nychka, D. and Naveau, P. (2007) Bayesian spatial modelling of extreme precipitation return levels *Journal of the American Statistical Association* **102**:479, 824–840.

Davison, A.C., Padoan, S.A. and Ribatet, M. Statistical Modelling of Spatial Extremes. Submitted.

Examples

```
## Generate realizations from the model
n.site <- 30
n.obs <- 50
coord <- cbind(lon = runif(n.site, -10, 10), lat = runif(n.site, -10 , 10))

gp.loc <- rgp(1, coord, "powexp", sill = 4, range = 20, smooth = 1)
gp.scale <- rgp(1, coord, "powexp", sill = 0.4, range = 5, smooth = 1)
gp.shape <- rgp(1, coord, "powexp", sill = 0.01, range = 10, smooth = 1)

locs <- 26 + 0.5 * coord[,"lon"] + gp.loc
scales <- 10 + 0.2 * coord[,"lat"] + gp.scale
shapes <- 0.15 + gp.shape

data <- matrix(NA, n.obs, n.site)
for (i in 1:n.site)
  data[,i] <- rgev(n.obs, locs[i], scales[i], shapes[i])

loc.form <- y ~ lon
scale.form <- y ~ lat
shape.form <- y ~ 1

hyper <- list()
hyper$sills <- list(loc = c(1,8), scale = c(1,1), shape = c(1,0.02))
hyper$ranges <- list(loc = c(2,20), scale = c(1,5), shape = c(1, 10))
hyper$smooths <- list(loc = c(1,1/3), scale = c(1,1/3), shape = c(1, 1/3))
hyper$betaMeans <- list(loc = rep(0, 2), scale = c(9, 0), shape = 0)
hyper$betaIcov <- list(loc = solve(diag(c(400, 100))),
                       scale = solve(diag(c(400, 100))),
                       shape = solve(diag(c(10), 1, 1)))

## We will use an exponential covariance function so the jump sizes for
## the shape parameter of the covariance function are null.
prop <- list(gev = c(2.5, 1.5, 0.2), ranges = c(0.7, 0.75, 0.9), smooths = c(0,0,0))
start <- list(sills = c(4, .36, 0.009), ranges = c(24, 17, 16), smooths
              = c(1, 1, 1),  beta = list(loc = c(26, 0.5), scale = c(10, 0.2),
                                shape = c(0.15)))

mc <- latent(data, coord, loc.form = loc.form, scale.form = scale.form,
             shape.form = shape.form, hyper = hyper, prop = prop, start = start,
             n = 500)
mc
```

`lmadogram` *Computes the lambda-madogram*

Description

Computes the lambda-madogram for max-stable processes.

Usage

```
lmadogram(data, coord, n.bins, xlab, ylab, zlab, n.lambda = 11, marge =
"emp", col = terrain.colors(50, alpha = 0.5), theta = 90, phi = 20,
border = NA, ...)
```

Arguments

`data`	A matrix representing the data. Each column corresponds to one location.
`coord`	A matrix that gives the coordinates of each location. Each row corresponds to one location.
`n.bins`	The number of bins to be used. If missing, pairwise lambda-madogram estimates will be computed.
`xlab,ylab,zlab`	The x-axis, y-axis and z-axis labels. May be missing.
`n.lambda`	Integer giving the number of lambda values.
`marge`	Character string. If 'emp', probabilities of non exceedances are estimated using the empirical CDF. If 'mle' (default), maximum likelihood estimates are used.
`col`	The colors used to emphasize the gradient of the lambda-madogram.
`theta,phi,border`	Options to be passed to the `persp` function.
`...`	Additional options to be passed to the `persp` function.

Details

Let $Z(x)$ be a stationary process. The λ-madogram is defined as follows:

$$\nu_\lambda(h) = \frac{1}{2}\mathrm{E}\left[|F^\lambda(Z(x+h)) - F^{1-\lambda}(Z(x))|\right]$$

Value

A graphic and (invisibly) a matrix with the lag distances, the λ-madogram estimate.

Author(s)

Mathieu Ribatet

References

Naveau, P., Guillou, A., Cooley, D. and Diebolt, J. (2009) Modelling Pairwise Dependence of Maxima in Space. To appear in *Biometrika*.

See Also

madogram, fmadogram

Examples

```
n.site <- 50
locations <- matrix(runif(2*n.site, 0, 10), ncol = 2)
colnames(locations) <- c("lon", "lat")

##Simulate a max-stable process - with unit Frechet margins
data <- rmaxstab(40, locations, cov.mod = "whitmat", sill = 1, range = 1,
smooth = 2)

##Compute the lambda-madogram
lmadogram(data, locations, n.bins = 80)
```

---

logLik *Extracts Log-Likelihood*

Description

Extract the pairwise log-likelihood for objects of class "maxstab"

Usage

```
## S3 method for class 'maxstab'
logLik(object, ...)
```

Arguments

object	An object of class "maxstab". Most often this will be the output of the fitmaxstab function.
...	Other arguments to be passed to the logLik function.

Value

Standard logLik object (see logLik) except that this is the pairwise log-likelihood!

Author(s)

Mathieu Ribatet

See Also

logLik

Examples

```
##Define the coordinates of each location
n.site <- 30
locations <- matrix(5 + runif(2*n.site, 0, 10), ncol = 2)

##Simulate a max-stable process - with unit Frechet margins
data <- rmaxstab(30, locations, cov.mod = "whitmat", sill = 1, range = 3,
smooth = 0.5)
fit <- fitmaxstab(data, locations, "whitmat")
logLik(fit)
```

`lsmaxstab` *Estimates the spatial dependence parameter of a max-stable process by*

Description

Estimates the spatial dependence parameter of a max-stable process by minimizing least squares.

Usage

```
lsmaxstab(data, coord, cov.mod = "gauss", marge = "emp", control =
list(), iso = FALSE, ..., weighted = TRUE)
```

Arguments

`data`	A matrix representing the data. Each column corresponds to one location.
`coord`	A matrix that gives the coordinates of each location. Each row corresponds to one location.
`cov.mod`	Character string specifying the max-stable process considered. Must be one of "gauss" (Smith's model), "whitmat", "cauchy", "powexp", "bessel", "caugen" for the Schlather model with the corresponding correlation function.
`marge`	Character string specifying how margins are transformed to unit Frechet. Must be one of "emp", "frech" or "mle" - see function `fitextcoeff`.
`control`	The control arguments to be passed to the `optim` function.
`iso`	Logical. If `TRUE`, isotropy is supposed. Otherwise (default), anisotropy is allowed. Currently this is only useful for the Smith model.
`...`	Optional arguments.
`weighted`	Logical. Should weighted least squares be used? See Details.

Details

The fitting procedure is based on weighted least squares. More precisely, the fitting criteria is to minimize:

$$\sum_{i,j} \left(\frac{\tilde{\theta}_{i,j} - \hat{\theta}_{i,j}}{s_{i,j}} \right)^2$$

where $\tilde{\theta}_{i,j}$ is a non parametric estimate of the extremal coefficient related to location `i` and `j`, $\hat{\theta}_{i,j}$ is the fitted extremal coefficient derived from the maxstable model considered and $s_{i,j}$ are the standard errors related to the estimates $\tilde{\theta}_{i,j}$.

Value

An object of class maxstab.

Author(s)

Mathieu Ribatet

References

Smith, R. L. (1990) Max-stable processes and spatial extremes. Unpublished manuscript.

See Also

fitcovariance, fitmaxstab, fitextcoeff

Examples

```
n.site <- 50
n.obs <- 100
locations <- matrix(runif(2*n.site, 0, 40), ncol = 2)
colnames(locations) <- c("lon", "lat")

## Simulate a max-stable process - with unit Frechet margins
data <- rmaxstab(50, locations, cov.mod = "gauss", cov11 = 200, cov12 =
0, cov22 = 200)

lsmaxstab(data, locations, "gauss")

##Force an isotropic model and do not use weights
lsmaxstab(data, locations, "gauss", iso = TRUE, weighted = FALSE)
```

---

madogram *Computes madograms*

Description

Computes the madogram for max-stable processes.

Usage

```
madogram(data, coord, fitted, n.bins, gev.param = c(0, 1, 0), which =
c("mado", "ext"), xlab, ylab, col = c(1, 2), angles = NULL, marge =
"emp", add = FALSE, xlim = c(0, max(dist)), ...)
```

Arguments

data	A matrix representing the data. Each column corresponds to one location.
coord	A matrix that gives the coordinates of each location. Each row corresponds to one location.
fitted	An object of class maxstab - usually the output of the fitmaxstab function. May be missing.

`n.bins`	The number of bins to be used. If missing, pairwise madogram estimates will be computed.
`gev.param`	Numeric vector of length 3 specifying the location, scale and shape parameters for the GEV.
`which`	A character vector of maximum size 2. It specifies if the madogram and/or the extremal coefficient functions have to be plotted.
`xlab,ylab`	The x-axis and y-axis labels. May be missing. Note that `ylab` must have the same length has `which`.
`col`	The colors used for the points and optionnaly for the fitted curve.
`angles`	A numeric vector. A partition of the interval $(0, \pi)$ to help detecting anisotropy.
`marge`	Character string. If 'emp', the observation are first transformed to the unit Frechet scale by using the empirical CDF. If 'mle' (default), maximum likelihood estimates are used.
`add`	Logical. If `TRUE`, the plot is added to the current figure; otherwhise (default) a new plot is computed.
`xlim`	A numeric vector of length 2 specifying the x coordinate range.
`...`	Additional options to be passed to the `plot` function.

Details

Let $Z(x)$ be a stationary process. The madogram is defined as follows:

$$\nu(h) = \frac{1}{2}\mathrm{E}\left[|Z(x+h) - Z(x)|\right]$$

If now $Z(x)$ is a stationary max-stable random field with GEV marginals. Provided the GEV shape parameter ξ is such that $\xi < 1$. The extremal coefficient $\theta(h)$ satisfies:

$$\theta(h) = \begin{cases} u_\beta\left(\mu + \frac{\nu(h)}{\Gamma(1-\xi)}\right), & \xi \neq 0 \\ \exp\left(\frac{\nu(h)}{\sigma}\right), & \xi = 0 \end{cases}$$

where $\Gamma(\cdot)$ is the gamma function and u_β is defined as follows:

$$u_\beta(u) = \left(1 + \xi\frac{u-\mu}{\sigma}\right)_+^{1/\xi}$$

and $\beta = (\mu, \sigma, \xi)$ i.e the vector of the GEV parameters.

Value

A graphic and (invisibly) a matrix with the lag distances, the madogram and extremal coefficient estimates.

Author(s)

Mathieu Ribatet

References

Cooley, D., Naveau, P. and Poncet, P. (2006) Variograms for spatial max-stable random fields. *Dependence in Probability and Statistics*, 373–390.

See Also

fmadogram, lmadogram

Examples

```
n.site <- 15
locations <- matrix(runif(2*n.site, 0, 10), ncol = 2)
colnames(locations) <- c("lon", "lat")

##Simulate a max-stable process - with unit Frechet margins
data <- rmaxstab(40, locations, cov.mod = "whitmat", sill = 1, range = 1,
smooth = 2)

##Compute the madogram
madogram(data, locations)

##Compare the madogram with a fitted max-stable model
fitted <- fitmaxstab(data, locations, "whitmat", sill = 1)
madogram(fitted = fitted, which = "ext")
```

---

map *Produces a 2D map from a fitted max-stable process*

Description

Produces a 2D map from a fitted max-stable process.

Usage

```
map(fitted, x, y, covariates = NULL, param = "quant", ret.per = 100, col
= terrain.colors(64), plot.contour = TRUE, ...)
```

Arguments

fitted	An object of class maxstab. Most often, it will be the output of the function fitmaxstab.
x,y	Numeric vector that gives the coordinates of the grid.
covariates	An array specifying the covariates at each grid point defined by x and y. If NULL, no covariate is needed. See the example to see how to build it.
param	A character string. Must be one of "loc", "scale", "shape" or "quant" for a map of the location, scale, shape parameters or for a map of a specified quantile.
ret.per	A numeric giving the return period for which the quantile map is plotted. It is only required if param = "quant".
col	A list of colors such as that generated by 'rainbow', 'heat.colors', 'topo.colors', 'terrain.colors' or similar functions.
plot.contour	Logical. If TRUE (default), contour lines are added to the plot.
...	Several arguments to be passed to the link{image} function.

Value

A plot. Additionally, a list with the details for plotting the map is returned invisibly.

Author(s)

Mathieu Ribatet

See Also

condmap, filled.contour, heatmap, heat.colors, topo.colors, terrain.colors, rainbow

Examples

```
##We run an artifical example using the volcano data set as a study
##region
dim <- dim(volcano)
n.x <- dim[1]
n.y <- dim[2]

x <- 10 * 1:n.x
y <- 10 * 1:n.y

n.site <- 15
idx.x <- sample(n.x, n.site)
idx.y <- sample(n.y, n.site)
locations <- cbind(lon = x[idx.x], lat = y[idx.y])
alt <- diag(volcano[idx.x, idx.y])

##Simulate a max-stable process - with unit Frechet margins
data <- rmaxstab(40, locations, cov.mod = "whitmat", sill = 1, range = 750,
smooth = 1)

##Now define the spatial model for the GEV parameters
param.loc <- -10 - 0.04 * locations[,1] + alt / 5
param.scale <- 5 - locations[,2] / 30 + alt / 4
param.shape <- rep(.2, n.site)

##Transform the unit Frechet margins to GEV
for (i in 1:n.site)
  data[,i] <- frech2gev(data[,i], param.loc[i], param.scale[i],
param.shape[i])

##Define a model for the GEV margins to be fitted
##shape ~ 1 stands for the GEV shape parameter is constant
##over the region
loc.form <- loc ~ lon + alt
scale.form <- scale ~ lat + alt
shape.form <- shape ~ 1

##  1- Fit a max-stable process
schlather <- fitmaxstab(data, locations, "whitmat", loc.form, scale.form,
                        shape.form, marg.cov = cbind(alt = alt), sill =
1, std.err.type = "none")

##  2- Produce a map of the pointwise 50-year return level

##Here we have only one covariate i.e. alt
n.cov <- 1
covariates <- array(volcano, dim = c(n.x, n.y, n.cov), dimnames =
```

```
list(NULL, NULL, "alt"))

par(mfrow = c(1,2))
image(x, y, volcano, col = terrain.colors(64), main = "Elevation map")
map(schlather, x, y, covariates, ret.per = 50, plot.contour = FALSE,
main = "50-year return level")
```

`map.latent` *Two dimensional map from a Bayesian hierarchical model*

Description

This function plots 2D maps from a Markov chain.

Usage

```
map.latent(fitted, x, y, covariates = NULL, param = "quant", ret.per =
100, col = terrain.colors(64), plot.contour = TRUE, fun = mean, level =
0.95, show.data = TRUE, control = list(nlines = 500), ...)
```

Arguments

`fitted`	An object of class "latent". Typically this will be the output of `latent`.
`x,y`	Numeric vector specifying the coordinates of the grid points.
`covariates`	An array specifying the covariates at each grid point defined by `x` and `y`. If `NULL`, no covariate is needed.
`param`	A character string. Must be one of "loc", "scale", "shape" or "quant" for a map of the location, scale, shape parameters or for a map of a specified quantile.
`ret.per`	A numeric giving the return period for which the quantile map is plotted. It is only required if `param = "quant"`.
`col`	A list of colors such as that generated by 'rainbow', 'heat.colors', 'topo.colors', 'terrain.colors' or similar functions.
`plot.contour`	Logical. If `TRUE` (default), contour lines are added to the plot.
`fun`	A character string specifying the function to be used to get posterior point estimates. The default is to take posterior means.
`level`	A numeric specifying the significance level for the pointwise credible intervals.
`show.data`	Logical. Should the locations where have observed the process have to be plotted?
`control`	A list with named components specifying options to be passed to `rgp`. Typically one might want specify `nlines` to reduce the computational demand.
`...`	Several arguments to be passed to the `link{image}` function.

Value

A plot and a invisible list containing all the data required to do the plot.

Author(s)

Mathieu Ribatet

See Also

condrgp, map

Examples

```
## Generate realizations from the model
n.site <- 30
n.obs <- 50
coord <- cbind(lon = runif(n.site, -10, 10), lat = runif(n.site, -10 , 10))

gp.loc <- rgp(1, coord, "powexp", sill = 4, range = 20, smooth = 1)
gp.scale <- rgp(1, coord, "powexp", sill = 0.4, range = 5, smooth = 1)
gp.shape <- rgp(1, coord, "powexp", sill = 0.01, range = 10, smooth = 1)

locs <- 26 + 0.5 * coord[,"lon"] + gp.loc
scales <- 10 + 0.2 * coord[,"lat"] + gp.scale
shapes <- 0.15 + gp.shape

data <- matrix(NA, n.obs, n.site)
for (i in 1:n.site)
  data[,i] <- rgev(n.obs, locs[i], scales[i], shapes[i])

loc.form <- y ~ lon
scale.form <- y ~ lat
shape.form <- y ~ 1

hyper <- list()
hyper$sills <- list(loc = c(1,8), scale = c(1,1), shape = c(1,0.02))
hyper$ranges <- list(loc = c(2,20), scale = c(1,5), shape = c(1, 10))
hyper$smooths <- list(loc = c(1,1/3), scale = c(1,1/3), shape = c(1, 1/3))
hyper$betaMeans <- list(loc = rep(0, 2), scale = c(9, 0), shape = 0)
hyper$betaIcov <- list(loc = solve(diag(c(400, 100))),
                       scale = solve(diag(c(400, 100))),
                       shape = solve(diag(c(10), 1, 1)))

## We will use an exponential covariance function so the jump sizes for
## the shape parameter of the covariance function are null.
prop <- list(gev = c(2.5, 1.5, 0.2), ranges = c(0.7, 0.75, 0.9), smooths = c(0,0,0))
start <- list(sills = c(4, .36, 0.009), ranges = c(24, 17, 16), smooths
              = c(1, 1, 1),  beta = list(loc = c(26, 0.5), scale = c(10, 0.2),
                                shape = c(0.15)))

## Generate a Markov chain
mc <- latent(data, coord, loc.form = loc.form, scale.form = scale.form,
             shape.form = shape.form, hyper = hyper, prop = prop, start = start,
             n = 100)

x.grid <- y.grid <- seq(-10, 10, length = 50)
map.latent(mc, x.grid, y.grid, param = "shape")
```

`margin fits` *Fits univariate extreme value distributions to data*

Description

These functions fit the generalised extreme value and generalised Pareto distribution to data using maximum likelihood.

Usage

```
gevmle(x, ..., method = "Nelder")
gpdmle(x, threshold, ..., method = "Nelder")
```

Arguments

`x`	Numeric vector of observations
`...`	Optional arguments to be passed to the `optim` function.
`threshold`	Numeric. The threshold value.
`method`	The numerical optimisation method to be used.

Details

These two functions are "extremely light" functions to fit the GEV/GPD. These functions are mainly useful to compute starting values for the Schlather and Smith model - see `fitmaxstab`.

If more refined (univariate) analysis have to be performed, users should use more specialised packages - e.g. POT, evd, ismev, ...

Value

A vector for the estimated parameters of the GEV/GPD.

Author(s)

Mathieu Ribatet

Examples

```
## 1 - GEV fit
x <- rep(NA, 100)
for (i in 1:100)
  x[i] <- max(rnorm(365))

gevmle(x)

## 2- GPD fit
x <- rnorm(10000)
##we need to fix a threshold
u <- quantile(x, 0.99)
gpdmle(x, u)
```

`modeldef` *Define a model for the spatial behaviour of the GEV parameters*

Description

This function defines the model for the spatial behaviour of the GEV parameter.

Usage

```
modeldef(data, formula)
```

Arguments

`data`	A matrix representing the data. Each column corresponds to one location.
`formula`	A R formula. See details for further details.

Value

need to be documented

Author(s)

Mathieu Ribatet

See Also

`formula`

Examples

```
## 1- A design matrix from a classical linear model
n.site <- 5
coord <- matrix(rnorm(2*n.site, sd = sqrt(.2)), ncol = 2)
colnames(coord) <- c("lon", "lat")
loc.form <- loc ~ lat + I(lon^2)
modeldef(coord, loc.form)

## 2- A design and penalization matrix from a penalized smoothin spline
x <- sort(runif(10, -2, 10))
n.knots <- 3
knots <- quantile(x, prob = 1:n.knots / (n.knots + 2))
modeldef(x, y ~ rb(x, knots = knots, degree = 3, penalty = 1))
```

`plot.maxstab` *Model checking of a fitted max-stable model*

Description

This function produces several plots to assess the goodness of fit of a fitted max-stable model.

Usage

```
## S3 method for class 'maxstab'
plot(x, ..., sites)
```

Arguments

`x`	An object of class `maxstab`. Most often, this will be the output of `fitmaxstab` or `lsmaxstab`.
`...`	Here for compatibility reasons but not yet implemented.
`sites`	A vector of integer of length 4 specifying the locations to be used for the model checking. If missing, locations will be choosen randomly.

Details

The diagonal plots are return level plots. The lower ones are qq-plots (on the Gumbel scale) between observed pairwise maxima for each block, e.g. year, and the ones obtained by simulations from the fitted model. The upper plot compares the fitted extremal coefficient functions to semi-empirical estimates from the F-madogram - see `fmadogram`. The two remaining plots are the stations locations and a qq-plot of blockwise maxima where the block size is 4.

Value

Several diagnostic plots.

Author(s)

Mathieu Ribatet

Examples

```
n.site <- 20
n.obs <- 50
coord <- matrix(runif(2 * n.site, 0, 10), ncol = 2)
colnames(coord) <- c("lon", "lat")
data <- rmaxstab(n.obs, coord, "powexp", sill = 1, range = 3, smooth =
1)
fitted <- fitmaxstab(log(data), coord, "powexp", y ~ 1, y ~ 1, y ~ 1,
sill = 1)
plot(fitted)
```

`predict.maxstab` *Prediction of the max-stable marginal parameters*

Description

This function predicts the marginal GEV parameters from a fitted max-stable process.

Usage

```
## S3 method for class 'maxstab'
predict(object, newdata, ret.per = NULL, std.err =
TRUE, ...)
```

Arguments

`object`	An object of class "maxstab". Most often, it will be the output of the function `fitmaxstab`.
`newdata`	An optional data frame in which to look for variables with which to predict. If omitted, the fitted values are used.
`ret.per`	Numeric vector giving the return periods for which return levels are computed. If `NULL` (default), no return levels are computed.
`std.err`	Logical. If `TRUE` (default), standard erros will be computed if possible.
`...`	further arguments passed to or from other methods.

Value

'predict.maxstab' produces a vector of predictions or a matrix of predictions.

Author(s)

Mathieu Ribatet

See Also

`predict`

Examples

```
## 1- Simulate a max-stable random field
n.site <- 35
locations <- matrix(runif(2*n.site, 0, 10), ncol = 2)
colnames(locations) <- c("lon", "lat")

data <- rmaxstab(50, locations, cov.mod = "whitmat", sill = 1, range = 30,
smooth = 0.5)

## 2- Transformation to non unit Frechet margins
param.loc <- -10 + 2 * locations[,2]
param.scale <- 5 + 2 * locations[,1]
param.shape <- rep(0.2, n.site)

for (i in 1:n.site)
```

```
  data[,i] <- frech2gev(data[,i], param.loc[i], param.scale[i],
param.shape[i])

## 3- Fit a max-stable process with the following model for
##    the GEV parameters
form.loc <- loc ~ lat
form.scale <- scale ~ lon
form.shape <- shape ~ 1

schlather <- fitmaxstab(data, locations, "whitmat", loc.form = form.loc,
                        scale.form = form.scale, shape.form =
                        form.shape)

## 4- GEV parameters estimates at each locations or at ungauged locations
predict(schlather)
ungauged <- data.frame(lon = runif(10, 0, 10), lat = runif(10, 0, 10))
predict(schlather, ungauged)
```

`predict.pspline` *Prediction of smoothing spline with radial basis functions*

Description

This function predicts the response from a fitted smoothing spline.

Usage

```
## S3 method for class 'pspline'
predict(object, new.data, ...)
```

Arguments

`object`	An object of class "pspline". Most often, it will be the output of the function `rbpspline`.
`new.data`	An optional data frame in which to look for variables with which to predict. If omitted, the fitted values are used.
`...`	further arguments passed to or from other methods.

Value

'predict.pspline' produces a vector of predictions or a matrix of predictions.

Author(s)

Mathieu Ribatet

See Also

`predict`

Examples

```
## 1- Define a function to approximate
fun <- function(x)
  sin(3 * pi * x)

## 2- Compute the response from fun - and adding a noise
x <- seq(0, 1, length = 200)
y <- fun(x) + rnorm(200, 0, sqrt(0.4))

## 2- Fit a penalized smoothing spline
n.knots <- 30
knots <- quantile(x, prob = 1:n.knots / (n.knots + 2))
fitted <- rbpspline(y, x, knots, degree = 3)

## 3- Prediction from the fitted spline
plot(x, y, pch = 2, col = "red")
plot(fun, from = 0, to = 1, add = TRUE)
pred <- predict(fitted)
lines(pred[,1], pred[,2], col = "blue", pch = 3)
```

---

predict.spatgev *Prediction of the GEV parameters*

Description

This function predicts the marginal GEV parameters from a fitted "spatial GEV" model.

Usage

```
## S3 method for class 'spatgev'
predict(object, newdata, ret.per = NULL, ...)
```

Arguments

object	An object of class spatgev". Most often, it will be the output of the function fitspatgev.
newdata	An optional data frame in which to look for variables with which to predict. If omitted, the fitted values are used.
ret.per	Numeric vector giving the return periods for which return levels are computed. If NULL (default), no return levels are computed.
...	further arguments passed to or from other methods.

Value

'predict.spatgev' produces a vector of predictions or a matrix of predictions.

Author(s)

Mathieu Ribatet

See Also

predict

Examples

```
## 1- Simulate a max-stable random field
n.site <- 35
locations <- matrix(runif(2*n.site, 0, 10), ncol = 2)
colnames(locations) <- c("lon", "lat")

data <- rmaxstab(50, locations, cov.mod = "whitmat", sill = 1, range = 3,
smooth = 0.5)
## 2- Transformation to non unit Frechet margins
param.loc <- -10 + 2 * locations[,2]
param.scale <- 5 + 2 * locations[,1]
param.shape <- rep(0.2, n.site)

for (i in 1:n.site)
  data[,i] <- frech2gev(data[,i], param.loc[i], param.scale[i],
param.shape[i])

## 3- Fit a ''spatial GEV'' mdoel to data with the following models for
##    the GEV parameters
form.loc <- loc ~ lat
form.scale <- scale ~ lon
form.shape <- shape ~ 1

fitted <- fitspatgev(data, locations, form.loc, form.scale, form.shape)

## 4- GEV parameters estimates at each locations or at ungauged locations
predict(fitted)
ungauged <- data.frame(lon = runif(10, 0, 10), lat = runif(10, 0, 10))
predict(fitted, ungauged)
```

---

print.latent *Printing objects of class "latent"*

Description

A method for printing object of class “maxstab”.

Usage

```
## S3 method for class 'latent'
print(x, digits = max(3, getOption("digits") - 3),
..., level = 0.95)
```

Arguments

x	An object of class “latent”. Most often, x is the output of the latent function.
digits	The number of digits to be printed.
...	Other options to be passed to the print function.
level	A numeric giving the significance level for the credible intervals.

Value

Print several information on screen.

Author(s)

Mathieu Ribatet

Examples

```
## Generate realizations from the model
n.site <- 15
n.obs <- 35
coord <- cbind(lon = runif(n.site, -10, 10), lat = runif(n.site, -10 , 10))

gp.loc <- rgp(1, coord, "powexp", sill = 4, range = 20, smooth = 1)
gp.scale <- rgp(1, coord, "powexp", sill = 0.4, range = 5, smooth = 1)
gp.shape <- rgp(1, coord, "powexp", sill = 0.01, range = 10, smooth = 1)

locs <- 26 + 0.5 * coord[,"lon"] + gp.loc
scales <- 10 + 0.2 * coord[,"lat"] + gp.scale
shapes <- 0.15 + gp.shape

data <- matrix(NA, n.obs, n.site)
for (i in 1:n.site)
  data[,i] <- rgev(n.obs, locs[i], scales[i], shapes[i])

loc.form <- y ~ lon
scale.form <- y ~ lat
shape.form <- y ~ 1

hyper <- list()
hyper$sills <- list(loc = c(1,8), scale = c(1,1), shape = c(1,0.02))
hyper$ranges <- list(loc = c(2,20), scale = c(1,5), shape = c(1, 10))
hyper$smooths <- list(loc = c(1,1/3), scale = c(1,1/3), shape = c(1, 1/3))
hyper$betaMeans <- list(loc = rep(0, 2), scale = c(9, 0), shape = 0)
hyper$betaIcov <- list(loc = solve(diag(c(400, 100))),
                       scale = solve(diag(c(400, 100))),
                       shape = solve(diag(c(10), 1, 1)))

## We will use an exponential covariance function so the jump sizes for
## the shape parameter of the covariance function are null.
prop <- list(gev = c(2.5, 1.5, 0.2), ranges = c(0.7, 0.75, 0.9),
 smooths = c(0,0,0))
start <- list(sills = c(4, .36, 0.009), ranges = c(24, 17, 16), smooths
              = c(1, 1, 1),  beta = list(loc = c(26, 0), scale = c(10, 0),
                             shape = c(0.15)))

mc <- latent(data, coord, loc.form = loc.form, scale.form = scale.form,
             shape.form = shape.form, hyper = hyper, prop = prop, start = start,
             n = 500)
mc
```

`print.maxstab` *Printing objects of class "maxstab"*

Description

A method for printing object of class “maxstab”.

Usage

```
## S3 method for class 'maxstab'
print(x, digits = max(3, getOption("digits") - 3),
...)
```

Arguments

`x`	An object of class “maxstab”. Most often, `x` is the output of the `fitmaxstab` function.
`digits`	The number of digits to be printed.
`...`	Other options to be passed to the `print` function.

Value

Print several information on screen.

Author(s)

Mathieu Ribatet

Examples

```
##Define the coordinates of each location
n.site <- 30
locations <- matrix(5 + rnorm(2*n.site, sd = sqrt(2)), ncol = 2)

##Simulate a max-stable process - with unit Frechet margins
data <- rmaxstab(30, locations, cov.mod = "whitmat", sill = 1, range = 3,
smooth = 0.5)

fit <- fitmaxstab(data, locations, "whitmat")
fit
```

print.pspline *Printing objects of class "pspline"*

Description

A method for printing object of class “pspline”.

Usage

```
## S3 method for class 'pspline'
print(x, ...)
```

Arguments

x	An object of class “pspline”. Most often, x is the output of the rbpspline function.
...	Other options to be passed to the print function.

Value

Print several information on screen.

Author(s)

Mathieu Ribatet

Examples

```
n <- 200
x <- runif(n)
fun <- function(x) sin(3 * pi * x)
y <- fun(x) + rnorm(n, 0, sqrt(0.4))
knots <- quantile(x, prob = 1:31 / 32)
fitted <- rbpspline(y, x, knots = knots, degree = 3)
fitted
```

print.spatgev *Printing objects of class "spatgev"*

Description

A method for printing object of class ”spatgev”.

Usage

```
## S3 method for class 'spatgev'
print(x, digits = max(3, getOption("digits") - 3),
...)
```

Arguments

x	An object of class "spatgev". Most often, x is the output of the fitspatgev function.
digits	The number of digits to be printed.
...	Other options to be passed to the print function.

Value

Print several information on screen.

Author(s)

Mathieu Ribatet

Examples

```
## 1- Simulate a max-stable random field
n.site <- 35
locations <- matrix(runif(2*n.site, 0, 10), ncol = 2)
colnames(locations) <- c("lon", "lat")

data <- rmaxstab(50, locations, cov.mod = "whitmat", sill = 1, range = 3,
smooth = 0.5)

## 2- Transformation to non unit Frechet margins
param.loc <- -10 + 2 * locations[,2]
param.scale <- 5 + 2 * locations[,1]
param.shape <- rep(0.2, n.site)

for (i in 1:n.site)
  data[,i] <- frech2gev(data[,i], param.loc[i], param.scale[i],
param.shape[i])

## 3- Fit a ''spatial GEV'' mdoel to data with the following models for
##    the GEV parameters
form.loc <- loc ~ lat
form.scale <- scale ~ lon
form.shape <- shape ~ 1

fitspatgev(data, locations, form.loc, form.scale, form.shape)
```

---

profile *Method for profiling fitted max-stable objects*

Description

Computes profile traces for fitted max-stable models.

Usage

```
## S3 method for class 'maxstab'
profile(fitted, param, range, n = 10, plot = TRUE,
conf = 0.90, method = "RJ", square = "chol", ...)
```

Arguments

`fitted`	An object of class "maxstab". Most often, it will be the output of the function `fitmaxstab`.
`param`	A character string giving the model parameter that are to be profiled.
`range`	The range for the profiled model parameter that must be explored.
`n`	Integer. The number of profiled model parameter that must be considered.
`plot`	Logical. If `TRUE` (default), the profile trace is plotted.
`conf`	Numeric giving the confidence interval level.
`method`	Character string. Must be one of "CB", "RJ" or "none" for the Chandler and Bate or the Rotnitzky and Jewell approaches respectively. The "none" method simply plots the profile of the log-composite likelihood. See details.
`square`	The choice for the matrix square root. This is only useful for the 'CB' method. Must be one of 'chol' (Cholesky) or 'svd' (Singular Value Decomposition).
`...`	Extra options that must be passed to the `plot` function.

Details

The Rotnitzky and Jewell approach consists in adjusting the distribution of the likelihood ratio statistics - which under misspecification is no longer χ^2 distributed.

The Chandler and Bate approach adjusts the composite likelihood itself is such a way that the usual asymptotic χ^2 null distribution is preserved. Note that in the current code, we use the singular value decomposition for the computation of matrix square roots to preserve asymmetry in the profile composite likelihood.

Value

A matrix. The first column corresponds to the values for which the profiled model parameter is fixed. The second column gives the value of the pairwise log-likelihood. The remaining columns contain the constrained maximum likelihood estimates for the remaining model parameters.

Warnings

This function can be **really** time consuming!

Author(s)

Mathieu Ribatet

References

Chandler, R. E. and Bate, S. (2007) Inference for clustered data using the independence loglikelihood *Biometrika*, **94**, 167–183.

Rotnitzky, A. and Jewell, N. (1990) Hypothesis testing of regression parameters in semiparametric generalized linear models for cluster correlated data. *Biometrika* **77**, 485–97.

Examples

```
## Not run:
##Define the coordinates of each location
n.site <- 30
locations <- matrix(rnorm(2*n.site, sd = sqrt(.2)), ncol = 2)
colnames(locations) <- c("lon", "lat")

##Simulate a max-stable process - with unit Frechet margins
data <- rmaxstab(50, locations, cov.mod = "gauss", cov11 = 100, cov12 =
25, cov22 = 220)

##Fit a max-stable process
##  1- using the Smith's model
fitted <- fitmaxstab(data, locations, "gauss", fit.marge = FALSE)

##Plot the profile pairwise log-likelihood for the ''cov11'' parameter
profile(fitted, "cov11", range = c(20, 180))

## End(Not run)
```

---

profile2d *Method for profiling (in 2d) fitted max-stable objects*

Description

Computes profile surfaces for fitted max-stable models.

Usage

```
## S3 method for class 'maxstab'
profile2d(fitted, params, ranges, n = 10, plot = TRUE,
...)
```

Arguments

`fitted`	An object of class "maxstab". Most often, it will be the output of the function `fitmaxstab`.
`params`	A character vector giving the two model parameters that are to be profiled.
`ranges`	A matrix corresponding to the ranges for the profiled model parameters that must be explored. Each row corresponds to one model parameter range.
`n`	Integer. The number of profiled model parameter that must be considered.
`plot`	Logical. If `TRUE` (default), the profile surface is plotted.
`...`	Extra options that must be passed to the `plot` function.

Value

A list with two arguments: `coord` and `llik`. `coord` is a matrix representing the grid where the profiled model parameters are fixed. `llik` the corresponding pairwise log-likelihood.

Warnings

This function can be **really** time consuming!

Author(s)

Mathieu Ribatet

Examples

```
## Not run:
##Define the coordinates of each location
n.site <- 30
locations <- matrix(rnorm(2*n.site, sd = sqrt(.2)), ncol = 2)
colnames(locations) <- c("lon", "lat")

##Simulate a max-stable process - with unit Frechet margins
data <- rmaxstab(30, locations, cov.mod = "whitmat", sill = 1, range = 30,
smooth = 0.5)

##Now define the spatial model for the GEV parameters
param.loc <- -10 + 2 * locations[,2]
param.scale <- 5 + 2 * locations[,1] + locations[,2]^2
param.shape <- rep(0.2, n.site)

##Transform the unit Frechet margins to GEV
for (i in 1:n.site)
  data[,i] <- frech2gev(data[,i], param.loc[i], param.scale[i],
param.shape[i])

##Define a model for the GEV margins to be fitted
##shape ~ 1 stands for the GEV shape parameter is constant
##over the region
loc.form <- loc ~ lat
scale.form <- scale ~ lon + (lat^2)
shape.form <- shape ~ 1

##Fit a max-stable process
##  1- using the Schlather representation
fitted <- fitmaxstab(data, locations, "whitmat", loc.form, scale.form,
                     shape.form)

##Plot the profile pairwise log-likelihood for the smooth parameter
ranges <- rbind(c(9,11), c(.3, .8))
profile2d(fitted, c("range", "smooth"), ranges = ranges)

## End(Not run)
```

---

qqextcoeff *QQ-plot for the extremal coefficient*

Description

This function compares the extremal coefficients estimated from a fitted max-stable process to the ones estimated semi-parametrically.

Usage

```
qqextcoeff(fitted, estim = "ST", marge = "emp", xlab = "Semi-Empirical",
ylab = "Model", ...)
```

Arguments

fitted	An object of class maxstab. Most often, this will be the output of fitmaxstab, fitcovmat or fitcovariance.
estim,marge	The estim and marge options to be passed to the fitextcoeff function.
xlab,ylab	The x and y-axis labels.
...	Optional arguments to be passed to the plot function.

Value

A QQ-plot.

Author(s)

Mathieu Ribatet

References

Schlather, M. (2002) Models for Stationary Max-Stable Random Fields. *Extremes* **5**:1, 33–44.

Schlather, M. and Tawn, J. A. (2003) A dependence measure for multivariate and spatial extreme values: Properties and inference. Biometrika **90**(1):139–156.

Smith, R. L. (1990) Max-stable processes and spatial extremes. Unpublished manuscript.

See Also

fitmaxstab, fitextcoeff

Examples

```
##Define the coordinate of each location
n.site <- 30
locations <- matrix(runif(2*n.site, 0, 10), ncol = 2)
colnames(locations) <- c("lon", "lat")

##Simulate a max-stable process - with unit Frechet margins
data <- rmaxstab(50, locations, cov.mod = "gauss", cov11 = 10, cov12 =
5, cov22 = 22)

fitted <- fitmaxstab(data, locations, "gauss")
qqextcoeff(fitted)
```

---

qqgev *QQ-plot for the GEV parameters*

Description

This function compares the GEV parameters estimated separately for each location to the ones estimated from a fitted spatial model.

Usage

```
qqgev(fitted, xlab, ylab, ...)
```

Arguments

fitted	An object of class maxstab or spatgev. Most often, this will be the output of fitmaxstab, fitcovmat, fitcovariance or fitspatgev.
xlab,ylab	The x and y-axis labels. May be missing.
...	Optional arguments to be passed to the plot function.

Value

A QQ-plot.

Author(s)

Mathieu Ribatet

References

Schlather, M. (2002) Models for Stationary Max-Stable Random Fields. *Extremes* **5**:1, 33–44.

Schlather, M. and Tawn, J. A. (2003) A dependence measure for multivariate and spatial extreme values: Properties and inference. Biometrika **90**(1):139–156.

Smith, R. L. (1990) Max-stable processes and spatial extremes. Unpublished manuscript.

See Also

qqextcoeff

Examples

```
##Define the coordinate of each location
n.site <- 30
locations <- matrix(runif(2*n.site, 0, 10), ncol = 2)
colnames(locations) <- c("lon", "lat")

##Simulate a max-stable process - with unit Frechet margins
data <- rmaxstab(50, locations, cov.mod = "gauss", cov11 = 100, cov12 =
25, cov22 = 220)

##Now define the spatial model for the GEV parameters
param.loc <- -10 + 2 * locations[,2]
param.scale <- 5 + 2 * locations[,1] + locations[,2]^2
param.shape <- rep(0.2, n.site)

##Transform the unit Frechet margins to GEV
for (i in 1:n.site)
  data[,i] <- frech2gev(data[,i], param.loc[i], param.scale[i], param.shape[i])

##Define a model for the GEV margins to be fitted
##shape ~ 1 stands for the GEV shape parameter is constant
##over the region
loc.form <- loc ~ lat
scale.form <- scale ~ lon + I(lat^2)
shape.form <- shape ~ 1

fitted <- fitspatgev(data, locations, loc.form = loc.form, scale.form =
scale.form, shape.form = shape.form)
```

```
qqgev(fitted)
```

`rainfall` *Summer annual maxima precipitation in Switzerland*

Description

Maximum daily rainfall amounts over the years 1962–2008 occuring during June–August at 79 sites in Switzerland.

Usage

```
data(rainfall)
```

Format

This data set contains two R objects: 'rain' and 'coord'. 'rain' is a 47 by 79 matrix giving the amount of rain in millimeters, each column correspond to one locations. 'coord' is a 79 by 3 matrix giving the longitude, latitude and the elevation for each station, all of them being in meters.

Author(s)

Mathieu Ribatet

Examples

```
data(rainfall)
op <- par(mfrow = c(1,2),pty = "s", mar = c(0,0,0,0))
swiss(city = TRUE)
idx.site <- c(1, 10, 20)
points(coord[-idx.site,])
points(coord[idx.site,], pch = 15, col = 2:4)

par(mar = c(2,4,0,0))
plot(1962:2008, rain[,1], type = "b", xlab = "Year", ylab =
"Precipitation (cm)", ylim = c(0, 120), col = 2)
lines(1962:2008, rain[,10], col = 3, type = "b")
lines(1962:2008, rain[,20], col = 4, type = "b")
par(op)
```

`rb` *Creates a model using penalized smoothing splines*

Description

Creates a model using penalized smoothing splines using radial basis functions

Usage

```
rb(..., knots, degree, penalty)
```

Arguments

...	The explicative variables for which the spline is based on.
knots	The coordinates of knots. See section details.
degree	Numeric. The degree of the spline.
penalty	Numeric. The penalty coefficient.

Details

If one explicative variable is given in "...", the knots should be a numeric vector. Otherwise, knots should be a matrix with the same number of column and covariates.

Value

A list giving all the required information to fit a penalized smoothing spline:

dsgn.mat	The design matrix.
pen.mat	The penalization matrix.
degree	The degree of the smoothing spline.
penalty	The penalty of the smoothing spline.
knots	The knots of the smoothing spline.
data	The explicative variables (e.g. covariates).
call	How was the rb function was called?

Warning

This function is not supposed to be called directly. rb is supposed to be embedded in a R formula.

Author(s)

Mathieu Ribatet

See Also

fitmaxstab

Examples

```
n.site <- 30
locations <- matrix(runif(2*n.site, 0, 10), ncol = 2)
colnames(locations) <- c("lon", "lat")
knots <- quantile(locations[,2], 1:5/6)

form <- y ~ rb(lat, knots = knots, degree = 3, penalty = .5)
```

`rbpspline` *Fits a penalized spline with radial basis functions to data*

Description

Fits a penalized spline with radial basis functions to data.

Usage

```
rbpspline(y, x, knots, degree, penalty = "gcv", ...)
```

Arguments

`y`	The response vector.
`x`	A vector/matrix giving the values of the predictor variable(s). If `x` is a matrix, each row corresponds to one observation.
`knots`	A vector givint the coordinates of the knots.
`degree`	The degree of the penalized smoothing spline.
`penalty`	A numeric giving the penalty coefficient for the penalization term. Alternatively, it could be either 'cv' or 'gcv' to choose the `penalty` using the (generalized) cross-validation criterion.
`...`	Additional options to be passed to the `cv` or `gcv` function.

Details

The penalized spline with radial basis is defined by:

$$f(x) = \beta_0 + \beta_1 x + \ldots + \beta_{m-1} x^{m-1} + \sum_{k=0}^{K-1} \beta_{m+k} ||x - \kappa_k||^{2m-1}$$

where β_i are the coefficients to be estimated, κ_i are the coordinates of the i-th knot and $m = \frac{d+1}{2}$ where d corresponds to the `degree` of the spline.

The fitting criterion is to minimize

$$||y - X\beta||^2 + \lambda^{2m-1} \beta^T K \beta$$

where λ is the penalty coefficient and K the penalty matrix.

Value

An object of class `pspline`.

Author(s)

Mathieu Ribatet

References

Ruppert, D. Wand, M.P. and Carrol, R.J. (2003) *Semiparametric Regression* Cambridge Series in Statistical and Probabilistic Mathematics.

See Also

cv, gcv

Examples

```
n <- 200
x <- runif(n)
fun <- function(x) sin(3 * pi * x)
y <- fun(x) + rnorm(n, 0, sqrt(0.4))
knots <- quantile(x, prob = 1:(n/4) / (n/4 + 1))
fitted <- rbpspline(y, x, knots = knots, degree = 3)
fitted

plot(x, y)
lines(fitted, col = 2)
```

---

rgp *Gaussian Random Fields Simulation*

Description

This functions generates gaussian random fields.

Usage

```
rgp(n, coord, cov.mod = "powexp", mean = 0, nugget = 0, sill = 1, range
= 1, smooth = 1, grid = FALSE, control = list())
```

Arguments

n	Integer. The number of replications.
coord	The locations coordinates for which the gaussian process is observed.
cov.mod	Character string. The covariance model used. Must be one of "whitmat", "cauchy", "powexp" of "cauchy". See the covariance function.
mean	Numeric. The mean of the gaussian random field.
nugget	Numeric. The nugget of the gaussian random field.
sill	Numeric. The sill parameter in the covariance function.
range	Numeric. The range parameter in the covariance function.
smooth	Numeric. The smooth parameter in the covariance function.
grid	Logical. Does coord defines a grid?
control	A named list with arguments 'nlines' (number of lines of the TBM simulation) and 'method' the name of the simulation method - must be one of 'exact', 'tbm' or 'circ'. If 'method' is NULL (default), the function tries to find the most appropriate simulation technique. If 'nlines' is NULL it is set to 1000.

Value

A matrix or an array containing the random field replicates.

Author(s)

Mathieu Ribatet

See Also

link{rmaxstab}

Examples

```
x <- y <- seq(0, 20, length = 100)
coord <- cbind(x, y)
gp <- rgp(1, coord, cov.mod = "whitmat", grid = TRUE)
filled.contour(x, y, gp, color.palette = terrain.colors)
```

---

rmaxstab *Simulation of Max-Stable Random Fields*

Description

This functions generates realisation from a max-stable random field.

Usage

```
rmaxstab(n, coord, cov.mod = "gauss", grid = FALSE, control =
list(), ...)
```

Arguments

n	Integer. The number of observations.
coord	A vector or matrix that gives the coordinates of each location. Each row corresponds to one location - if any.
cov.mod	A character string that gives the max-stable model. This must be one of "gauss" for the Smith model or "whitmat", "cauchy", "powexp" and "bessel" for the Schlather model with the given correlation family.
grid	Logical. Does the coordinates represent grid points?
control	A named list with arguments 'nlines' (number of lines of the TBM simulation), 'method' the name of the simulation method - must be one of 'exact', 'tbm' or 'circ', and 'uBound' the uniform upper bound. See details.
...	The parameters of the max-stable model. See details.

Details

Users must supply the parameters for the max-stable model. For the Schlather model, users should supply the "sill", "range" and "smooth" parameter values. For the Smith model, if coord is univariate you must specify var, otherwise users should supply the covariance parameters i.e. parameters with names such as cov11, cov12, ...

Here are the details for each allowed components of 'control'. If 'method' is NULL (default), the function tries to find the most appropriate simulation technique. Current simulation techniques are a direct approach, i.e. Cholesky decomposition of the covariance matrix, the turning bands and the circular embedding methods. If 'nlines' is NULL, it is set to 1000. If 'uBound' is NULL, it is set to reasonable values - for example 3.5 for the Schlather model.

Value

A matrix containing observations from the required max-stable model. Each column represents one stations. If `grid = TRUE`, the function returns an array of dimension nrow(coord) x nrow(coord) x n.

Author(s)

Mathieu Ribatet

References

Schlather, M. (2002) Models for Stationary Max-Stable Random Fields. *Extremes* **5**:1, 33–44.

Smith, R. L. (1990) Max-stable processes and spatial extremes. Unpublished manuscript.

See Also

`fitmaxstab`

Examples

```
## 1. Smith's model
set.seed(8)
x <- y <- seq(0, 10, length = 100)
coord <- cbind(x, y)
data <- rmaxstab(1, coord, "gauss", cov11 = 9/8, cov12 = 0, cov22 = 9/8,
  grid = TRUE)
##We change to unit Gumbel margins for visibility
filled.contour(x, y, log(data), color.palette = terrain.colors)

## 2. Schlather's model
data <- rmaxstab(1, coord, cov.mod = "powexp", sill = 1, range = 3,
smooth = 1, grid = TRUE)
filled.contour(x, y, log(data), color.palette = terrain.colors)
```

`SpatialExtremes` *Analysis of Spatial Extremes*

Description

The package **SpatialExtremes** aims to provide tools for the analysis of spatial extremes. Currently, the package uses the max-stable processes framework for the modelling of spatial extremes.

Max-stable processes are the extension of the extreme value theory to random fields. Consequently, they are good candidate to the analysis of spatial extremes. The strategy used in this package is to fit max-stable processes to data using composite likelihood.

In the future, the package will allow for non-stationarity as well as other approaches to model spatial extremes; namely latent variable and copula based approaches.

A package vignette has been writen to help new users. It can be viewed, from the R console, by invoking `vignette("SpatialExtremesGuide")`.

Details

The package provides the following main tools:

1. rgp: simulates gaussian random fields,
2. rmaxstab: simulates max-stable random fields,
3. fitspatgev: fits a spatial GEV model to data,
4. fitmaxstab, lsmaxstab: fits max-stable processes to data,
5. latent: draws a Markov chain from a Bayesian hierarchical model for spatial extremes,
6. predict: allows predictions for fitted max-stable processes,
7. map, condmap: plot a map for GEV parameter as well as return levels - or conditional return levels
8. anova, TIC, DIC: help users in model selection,
9. madogram, fmadogram, lmadogram: are (kind of) variograms devoted to extremes,
10. fitextcoeff: estimates semi-parametrically the extremal coefficient,
11. extcoeff: plots the evolution of the extremal coefficient from a fitted max-stable process,
12. rbpspline: fits a penalized spline with radial basis function,
13. gev2frech, frech2gev: transform GEV (resp. Frechet) observation to unit Frechet (resp. GEV) ones
14. gevmle, gpdmle: fit the GEV/GPD distributions to data,
15. distance: computes the distance between each pair of locations,
16. profile, profile2d: computes the profile composite likelihood,
17. covariance, variogram: computes the covariance/semivariogram function.

Acknowledgement

The development of the package has been financially supported by the Competence Center Environment and Sustainability (CCES) and more precisely within the EXTREMES project (http://www.cces.ethz.ch/projects/hazri/EXTREMES).

Author(s)

Mathieu Ribatet

---

swiss *Map of the Switzerland.*

Description

Plot a map of Switzerland and optionnaly some cities.

Usage

```
swiss(city = FALSE, add = FALSE, axes = FALSE, km = TRUE, xlab = "",
ylab = "", ...)
```

Arguments

city	Logical. It TRUE, some city are displayed. Default is to omit.
add	Logical. Should the map be added to an existing plot?
axes	Logical. Should the axis be displayed?
km	Logical. If TRUE (default) the longitude and latitude are expressed in kilometers. Otherwise it is in meters.
xlab,ylab	The x and y-axis labels.
...	Optional arguments to be passed to the plot function.

Value

A graphic window.

Author(s)

Dominik Schaub

Examples

```
swiss()
```

---

swissalt *Elevation in Switzerland*

Description

Data required for plotting a Switzerland map with elevation.

Usage

```
data(swissalt)
```

Format

This data set contains three R objects. 'alt.mat' is a 192 by 115 matrix giving the elevation at the grid points defined by 'lon.vec' and 'lat.vec'.

Author(s)

Mathieu Ribatet

Examples

```
data(swissalt)

layout(matrix(c(1,2), 1), width = c(3.5,1))
mar <- rep(0, 4)
op <- par(mar = mar)
breaks <- seq(0, 2000, by = 250)
image(lon.vec, lat.vec, alt.mat, col = terrain.colors(length(breaks) - 1),
      xaxt = "n", yaxt = "n", bty = "n", xlab = "", ylab = "", xlim =
```

```
c(480, 840), ylim = c(58, 300))
swiss(add = TRUE, city = TRUE)

##Heat bar
mar <- c(3, 3, 3, 4)
par(las = 1, mar = mar)

plot.new()
plot.window(xlim = c(0, 1), ylim = range(breaks), xaxs = "i",
            yaxs = "i")
rect(0, breaks[-length(breaks)], 1, breaks[-1], col = terrain.colors(length(breaks) - 1),
     border = NA)
axis(4, at = breaks[-length(breaks)])
box()
title("Elevation\n(meters)")
par(op)
```

`TIC` *Takeuchi's information criterion*

Description

Computes a the Takeuchi's information criterion which is equivalent to the AIC when the model is miss-specified.

Usage

```
TIC(object, ..., k = 2)
```

Arguments

`object`	An object of class `maxstab` or `spatgev`. Often, it will be the output of the `fitmaxstab` or `fitspatgev` function.
`...`	Additional objects of class `maxstab` or `spatgev` for which TIC should be computed.
`k`	Numeric. The penalty per parameter to be used. The case k = 2 (default) correspond to the classical TIC and $k = \log n$, n number of observations, is the robust version of the BIC.

Details

TIC is like AIC so that when comparing models one wants to get the lowest TIC score.

Value

Numeric.

Author(s)

Mathieu Ribatet

References

Gao, X. and Song, P. X.-K. (2009) Composite likelihood Bayesian information criteria for model selection in high dimensional data. Preprint.

Sakamoto, Y., Ishiguro, M., and Kitagawa G. (1986) Akaike Information Criterion Statistics. D. Reidel Publishing Company.

Varin, C. and Vidoni, P. (2005) A note on composite likelihood inference and model selection. Biometrika **92**(3):519–528.

See Also

fitmaxstab, AIC

Examples

```
##Define the coordinate of each location
n.site <- 50
locations <- matrix(runif(2*n.site, 0, 100), ncol = 2)
colnames(locations) <- c("lon", "lat")

##Simulate a max-stable process - with unit Frechet margins
data <- rmaxstab(40, locations, cov.mod = "whitmat", sill = 0.8, range =
30, smooth = 0.5)

M0 <- fitmaxstab(data, locations, "powexp", std.err.type = "score",
                 fit.marge = FALSE)
M1 <- fitmaxstab(data, locations, "cauchy", std.err.type = "score",
                 fit.marge = FALSE)

TIC(M0, M1)
TIC(M0, M1, k = log(nrow(data)))
```

---

variogram *Empirical variogram*

Description

This function computes the empirical variogram.

Usage

```
variogram(data, coord, n.bins, xlab, ylab, angles = NULL, add = FALSE,
xlim = c(0, max(dist)), ...)
```

Arguments

data	A matrix representing the data. Each column corresponds to one location.
coord	A matrix that gives the coordinates of each location. Each row corresponds to one location.
n.bins	The number of bins to be used. If missing, pairwise madogram estimates will be computed.

xlab,ylab	The x-axis and y-axis labels. May be missing. Note that ylab must have the same length has which.
angles	A numeric vector. A partition of the interval $(0, \pi)$ to help detecting anisotropy.
add	Logical. If TRUE, the plot is added to the current figure; otherwhise (default) a new plot is computed.
xlim	A numeric vector of length 2 specifying the x coordinate range.
...	Additional options to be passed to the plot function.

Value

A graphic and (invisibly) a matrix with the lag distances and the empirical variogram estimates.

Author(s)

Mathieu Ribatet

See Also

fmadogram, lmadogram

Examples

```
n.site <- 20
n.obs <- 100
coord <- matrix(runif(2 * n.site, 0, 10), ncol = 2)
data <- rgp(n.obs, coord, "powexp", sill = 2, range = 3, smooth = 1)
variogram(data, coord)
```

---

vdc *Van der Corput Sequence*

Description

This function generates the three dimensional Van der Corput sequence on the half unit Sphere of R^3 - eventually randomly rotated.

Usage

```
vdc(n, rand.rot = FALSE)
```

Arguments

n	Integer. The length of the sequence.
rand.rot	Logical. Should the sequence be randomly rotated?

Value

A matrix giving the coordinates of the points in the half unit sphere.

Author(s)

Mathieu Ribatet

References

Freulon, X., de Fouquet, C., 1991. Remarques sur la pratique des bandes tournantes a trois dimensions. Cahiers de geostatistique, Fascicule 1, Centre de Geostatistique, Ecole des Mines de Paris, Fontainebleau, pp. 101–117.

Examples

```
vdc(10)
```

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